Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Supplementary Examination, July- **2023**
APPLIED MATHEMATICS PAPER CODE 465

General Instructions: -

1 You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2 **“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”**

3 Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**

4 The Marking scheme carries only suggested value points for the answers.
These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.

5 The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

6 Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X” be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

7 If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

8 If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9 **In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.**

10 **In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–241)
(PAPER CODE: 465)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If $100 \equiv x \pmod{7}$, then the least positive value of x is : (a) 6 (b) 4 (c) 3 (d) 2	
Ans.	(d) 2	**1**
2.	In a kilometre race, A beats B by 50 metres or 10 seconds. The time taken by A to complete the race is : (a) 90 seconds (b) 120 seconds (c) 190 seconds (d) 200 seconds	
Ans.	(c) 190 seconds	**1**
3.	If a man rows 32 km downstream and 14 km upstream in 6 hours each, then the speed of the stream is : (a) 2 km/h (b) 1·5 km/h (c) 2·5 km/h (d) 2·25 km/h	
Ans.	(b) 1.5 km/h	**1**
4.	If $-3x + 17 < -13$, then : (a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10]$	

Ans.	(a) $\boldsymbol{x \in (10, \infty)}$	**1**
5.	If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to : (a) B (b) A (c) I (d) O	
Ans.	(a) B	**1**
6.	If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ is a symmetric matrix, then : (a) $x = 0, y = 5$ (b) $x = 5, y = 0$ (c) $x = y$ (d) $x + y = 0$	
Ans.	(c) $\boldsymbol{x = y}$	**1**
7.	The value of x for which the points $(2, -1)$, $(-3, 4)$ and $(x, 5)$ are collinear, is : (a) -4 (b) 4 (c) 2 (d) -2	
Ans.	(a) $\boldsymbol{-4}$	**1**
8.	If $x + y = 8$, then the maximum value of xy is : (a) 12 (b) 16 (c) 20 (d) 24	
Ans.	(b) 16	**1**

9.	The function $f(x) = x^x$, $x > 0$ is decreasing in the interval : (a) $(-\infty, e)$ (b) $(0, e)$ (c) $\left(0, \frac{1}{e}\right)$ (d) $\left[\frac{1}{e}, \infty\right)$	
Ans.	(c) $\left(\mathbf{0}, \frac{\mathbf{1}}{\boldsymbol{e}}\right)$	**1**
10.	$\int \frac{1}{x + x \log x} dx$ is equal to : (a) $1 + \log x + C$ (b) $x + \log x + C$ (c) $x \log (1 + \log x) + C$ (d) $\log (1 + \log x) + C$	
Ans.	(d) $\mathbf{\log(1 + \log \boldsymbol{x}) + \boldsymbol{C}}$	**1**
11.	If the supply function is $p = 4 + x$, then the producer's surplus when 12 units are sold, is : (a) 72 (b) 64 (c) 76 (d) 46	
Ans.	(a) 72	**1**
12.	The order of the differential equation corresponding to family of curves $y = Ae^{3x} + Be^{-3x}$ is : (a) 1 (b) 2 (c) 3 (d) 4	
Ans.	(b) 2	**1**
13.	If 'm' is the mean of Poisson distribution, then $P(r = 0)$ is given by : (a) e^{-m} (b) e^{m} (c) e (d) m^{-e}	

Ans.	(a) $\boldsymbol{e^{-m}}$	**1**
14.	Normal distribution is symmetric about : (a) variance (b) standard deviation (c) mean (d) covariance	
Ans.	(c) mean	**1**
15.	For a student's t-test, the test statistic t is given by : (a) $t = \overline{x} - \mu$ (b) $t = \frac{\overline{x}}{s}$ (c) $t = \frac{\overline{x} - \mu}{s}$ (d) $t = \frac{\frac{\overline{x} - \mu}{s}}{\sqrt{n-1}}$	
Ans.	(d) $\boldsymbol{t = \frac{\frac{\bar{x}-\mu}{s}}{\sqrt{n-1}}}$	**1**
16.	For the given values 15, 23, 28, 36, 41, 46, the 3-yearly moving averages are : (a) 24, 29, 35, 41 (b) 22, 28, 35, 41 (c) 22, 29, 35, 41 (d) 24, 28, 35, 41	
Ans.	(c) 22, 29, 35, 41	**1**
17.	The effective rate of return which is equivalent to nominal rate of 8% p.a. compounded quarterly is : [Given $(1{\cdot}02)^4 = 1{\cdot}0824$] (a) 8·16% (b) 7·95% (c) 8·24% (d) 8·5%	
Ans.	(c) 8.24 %	**1**

18.	The maximum value of the function $z = 7x + 5y$, subject to constraints $x \le 3, y \le 2, x \ge 0, y \ge 0$, is : (a) 21 (b) 10 (c) 31 (d) 37	
Ans.	(c) 31	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	
19.	*Assertion (A) :* The slope of the normal to the curve $y = 2x^2 - 5x$ at $x = -1$ is -1. *Reason (R) :* The slope of the normal to the curve $y = f(x)$ at point (α, β) is given by $(x - \alpha) + \left(\frac{dy}{dx}\right)_{(\alpha, \beta)} \cdot (y - \beta) = 0$.	
Ans.	Both (A) and (R) are false, so 1 mark may be given to all who attempted	**1**
20.	*Assertion (A) :* If A is a square matrix of order 3 such that $\lvert \text{adj } A \rvert = 144$, then the value of $\lvert A \rvert$ is ± 12. *Reason (R) :* If A is an invertible matrix of order n, then $\lvert \text{adj } A \rvert = \lvert A \rvert^{n-1}$.	
Ans.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises very short answer (VSA) type questions of 2 marks each.**	
21(a).	Solve the inequality : $\frac{3}{5}x - \frac{2x-1}{3} > 1, x \in W$	

Ans.	$\frac{3}{5}x-\frac{2x-1}{3}>1$ $\Rightarrow 9x-5(2x-1)>15$ Solving we get, $x<-10$ Solution set is $\emptyset$	1 ½ ½
	OR	
21(b).	Solve the inequality : $$-\frac{2}{3}<-\frac{x}{3}+1\le\frac{2}{3}, x\in R$$	
Ans.	$-\frac{2}{3}<-\frac{x}{3}+1\le\frac{2}{3}$ $\Rightarrow -\frac{5}{3}<-\frac{x}{3}\le-\frac{1}{3}$ $\Rightarrow \frac{1}{3}\le\frac{x}{3}<\frac{5}{3}$ $\Rightarrow 1\le x<5$ Solution set is $[1,5)$	1 1
22.	Write the matrix $\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.	
Ans.	Let $A=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\Rightarrow A'=\begin{bmatrix}7 & -1 & -1\\ -3 & 1 & 0\\ -3 & 0 & 1\end{bmatrix}$ $P=\frac{A+A'}{2}=\begin{bmatrix}7 & -2 & -2\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$ and $Q=\frac{A-A'}{2}=\begin{bmatrix}0 & -1 & -1\\ 1 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$ So, $P'=P\Rightarrow$ symmetric, $Q'=-Q\Rightarrow$ skew symmetric $A=\frac{A+A'}{2}+\frac{A-A'}{2}=\begin{bmatrix}7 & -2 & -2\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -1 & -1\\ 1 & 0 & 0\\ 1 & 0 & 0\end{bmatrix}$	½ ½+½ ½
23(a).	It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains no defective screw. (Given : $e^{-2}=0\cdot14$)	
Ans.	Let p be the probability that the screw is defective	

	Then $p = \frac{2}{100}$. Here $n = 100$ So, $m = np = 2$ Let X denote the number of defective screws in a packet of 100 screws. Then X follows the probability distribution as $P(X = r) = e^{-m}\frac{m^r}{r!}, r = 0,1,2 \ldots$ P(no defective screw) $= P(X = 0) = e^{-m}\frac{m^0}{0!} = e^{-2} = 0.14$	**1** **1**
	OR	
23(b).	If the standard deviation of a Poisson variable X is $\sqrt{3}$, then find P (X > 0). [Use : $e^{-3} = 0{\cdot}05$]	
Ans.	Let X follows Poisson distribution with parameter m Then Variance $= m = (\sqrt{3})^2 = 3$ $\therefore P(X = r) = e^{-m}\frac{m^r}{r!} = e^{-3}\frac{3^r}{r!}, r = 0,1,2 \ldots$ Hence $P(X > 0) = 1 - P(X = 0) = 1 - e^{-3} = 1 - 0.05 = 0.95$	**½** **1½**
24.	Find the present value of a sequence of payments of ₹ 1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.	
Ans.	The given annuity is a perpetuity of first type in which $R =$ ₹ 1000 and $r = \frac{8}{2}\% = 4\%$ per half year So, $i = \frac{4}{100} = 0.04$ Present value $= P = \frac{R}{i} = \frac{1000}{0.04} = 25000$ Hence the present value is ₹ 25000	**1** **1**
25.	The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is ₹ 97,200, find. (i) its value after 3 years; (ii) its value when it was purchased.	
Ans.	Given $P =$ ₹ $97{,}200, r = 10\%$ p.a $\Longrightarrow i = \frac{10}{100} = 0.1$ (i) Value after 3 years = Present value × $(1 - 0.1)^3$	

	$= ₹\ 97200(0.9)^3$ $= ₹\ 70858.80$ (ii) Present value = value 2 years ago × $(1-0.1)^2$ $\Longrightarrow 97200 =$Value 2 years ago $\times\ (0.9)^2$ $\Longrightarrow$ Value 2 years ago = ₹ $\frac{97200}{(0.9)^2} = ₹\ 120000$	**1** **1**
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	Formulate the following problem as an LPP : A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit.	
Ans.	Let $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the number of rings and chains respectively Maximize $\boldsymbol{Z = 300\ x + 190\ y}$ Subject to constraints $\left.\begin{array}{l} \boldsymbol{x+y \le 24} \\ \boldsymbol{x+\frac{y}{2} \le 16} \\ \boldsymbol{x, y \ge 0} \end{array}\right\}$	**1** **2**
27.	A person can row a boat at 5 km/h in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.	
Ans.	Let the rate at which the stream is flowing be x km/h and let the distance covered by the boat be y km According to question, $\frac{3y}{5+x} = \frac{y}{5-x}$ $\Longrightarrow 3(5-x) = 5+x$ $\Longrightarrow x = 2.5$ Thus, the stream is flowing at the rate of 2.5 km/h	**2** **1**

28(a).	A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated four more times. What is the amount of juice left in the container after final replacement ? [Take $(0{\cdot}9)^5 = 0{\cdot}59049$]	
Ans.	Total juice in the container = 50 litres Juice taken out = 5 litres No. of times process repeated = 5 Amount of juice in container after final replacement $= \mathbf{50}\left(\mathbf{1}-\frac{\mathbf{5}}{\mathbf{50}}\right)^{\mathbf{5}}$ $= \mathbf{29.52}$ litres	2 1
28(b).	In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C ?	
Ans.	A beats B by 100 metres, means A travels 1000 metres in the same time in which B travels 900 metres. B beats C by 100 metres, means B travels 1000 metres in the same time in which C travels 900 metres. $\left.\begin{array}{l}\therefore A : B = 10 : 9 \\ B : C = 10 : 9\end{array}\right\}$ $\Rightarrow \mathrm{A : B : C} = 100 : 90 : 81$ So, A travels 100 metres and in the same time C travels 81 metres Thus, A beats C by 190 metres	1 1 1
29(a).	A fair coin is tossed 9 times. Find the probability of getting. (i) exactly 5 tails; (ii) at least 5 tails; (iii) at most 5 tails.	

Ans.

Repeated tosses of a fair coin qualify as Bernoulii's trials

Let X denote the number of trails in an experiment of such trials and hence is the binomial distribution

Here $n = 9, p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

(a) $P(\text{exactly 5 success}) = P(X = 5) = 9_{C_5} p^5 q^4 = 9_{C_5} \left(\frac{1}{2}\right)^9$

$$= \frac{63}{256}$$

1

(b) $P(\text{at least 5 successes}) = P(X \geq 5)$

$$= \left(\frac{1}{2}\right)^9 \left[9_{C_5} + 9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}\right]$$

$$= \frac{256}{512} = \frac{1}{2}$$

1

(c) $P(\text{at most 5 successes}) = P(X \leq 5) = 1 - P(X > 5)$

$$= 1 - \left(\frac{1}{2}\right)^9 \left[9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}\right]$$

$$= 1 - \frac{130}{512} = \frac{382}{512} = \frac{191}{256}$$

1

OR

29(b).

Let X denote the number of hours a person watches T.V. during a randomly selected day. The probability that X can take the values x_i, has the following form, where k is some unknown constant.

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) Find : $P(X = 2)$, $P(X \geq 2)$ and $P(X \leq 2)$.

Ans.

$\boldsymbol{x_i}$	0	1	2	3	otherwise
$\boldsymbol{P(x_i) = p_i}$	0.2	$\boldsymbol{k}$	$\boldsymbol{2k}$	$\boldsymbol{2k}$	0

½

(i) As $\sum \boldsymbol{p_i} = \mathbf{1}$, we have

$$\mathbf{0.2 + k + 2k + 2k = 1}$$

$$\mathbf{\Rightarrow 5k = 0.8 \ \Rightarrow k = 0.16}$$

1

	(ii) $P(X = 2) = 2k = 0.32$ $P(X \geq 2) = 2k + 2k = 0.64$ $P(X \leq 2) = 0.2 + k + 2k = 0.68$	½ ½ ½
30.	2000 students appeared in an examination. Distribution of marks is assumed to be normal with mean 30 and standard deviation 6·25. How many students are expected to get marks (i) between 20 and 40 ? (ii) less than 25 ? [Use : $P(0 \leq z \leq 1{\cdot}6) = 0{\cdot}4452$, $P(0 \leq z \leq 0{\cdot}8) = 0{\cdot}2881$]	
Ans.	Let X denote the marks of the student. Mean $(\mu) = 30$, S.D $(\sigma) = 6.25$ $Z = \frac{X - \mu}{\sigma} = \frac{X - 30}{6.25}$ (i) When $X = 20, Z = -1.60$ When $X = 40, Z = 1.60$ $\therefore P(20 \leq X \leq 40) = P(-1.60 \leq Z \leq 1.60)$ $= P(-1.60 \leq Z \leq 0) + P(0 \leq Z \leq 1.60)$ $= 2P(0 \leq Z \leq 1.60)$ $= 2 \times 0.4452 = 0.8904$ Thus, out of 2000 students, the expected number of students getting marks between 20 and 40 = $2000 \times 0.8904 = 1780.8$ or 1781 (ii) When $X = 25, Z = -0.80$ So, $P(X < 25) = P(Z < -0.8) = P(Z > 0.8)$ $= P(Z \geq 0) - P(0 \leq Z \leq 0.8)$ $= 0.5 - 0.2881 = 0.2119$ Thus, out of 2000 students the expected number of students getting marks less than 25 = $2000 \times 0.2119 = 423.8$ or 424	½ ½ 1 1
31.	The mean of IQs of a group of 18 students from one area of a city was found to be 95 with a standard deviation of 10, while the mean of IQs of a group of 12 students from another area of the city was found to be 100 with a standard deviation of 8. Test whether there is a significant difference between the IQs of two groups of students at 1% level of significance. [Use : $t_{28}(0{\cdot}01) = 2{\cdot}763$]	
Ans.	Here $n_1 = 18, \overline{x} = 95, s_1 = 10$ and $n_2 = 12, \overline{y} = 100, s_2 = 8$	

	$\therefore s = \sqrt{\frac{n_1 {s_1}^2 + n_2 {s_2}^2}{n_1+n_2-2}} = \sqrt{\frac{18\times10^2+12\times8^2}{18+12-2}}$ $= \sqrt{\frac{2568}{28}} = 9.58$ Let H_0= no significant difference between IQs of two group of students H_α= significant difference between IQs of two group of students The test statistic $'t'$ is $t = \frac{\bar{x}-\bar{y}}{s} \times \sqrt{\frac{n_1 n_2}{n_1+n_2}}$ $= \frac{95-100}{9.58} \times \sqrt{\frac{18\times12}{18+12}} = -\frac{5}{9.58} \times 2.68 = -1.398$ $\Rightarrow \lvert t \rvert = 1.398$ The test statistics t follows student's t-distribution with $n = 18 + 12 - 2$ i.e., 28 degrees of freedom At 1% level of significance, we have $t_{28}(0.01) = 2.763$ As $1.398 < 2.763$, null hypothesis is accepted Hence, there is no significance difference between IQs of two groups	1½ 1 ½
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	Determine for what values of x, the function $f(x) = x^4 - \frac{x^3}{3}$ is strictly increasing or strictly decreasing.	
Ans.	$f(x) = x^4 - \frac{x^3}{3} \Rightarrow f'(x) = 4x^3 - x^2 = x^2(4x-1)$ $f'(x) = 0 \Rightarrow x = 0, \frac{1}{4}$ Thus, critical points divide R into three parts $x < 0, 0 < x < \frac{1}{4}, x > \frac{1}{4}$ When $x < 0, f'(x)$ is $-$ve $\therefore f(x)$ is strictly decreasing for $x < 0$ When $0 < x < \frac{1}{4}, f'(x)$ is +ve $\therefore f(x)$ is strictly increasing for $x < 0$ When $x > \frac{1}{4}, f'(x)$ is $-$ve $\therefore f(x)$ is strictly decreasing for $x < 0$ Hence $f(x)$ is strictly increasing on $\left(\frac{1}{4}, \infty\right)$ and strictly decreasing on $(-\infty, 0) \cup \left(0, \frac{1}{4}\right)$	1 ½ 1 1 1 ½

	OR	
32(b).	A firm has the following total cost and demand functions : $C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$ (i) Find the total revenue function in terms of x. (ii) Find the total profit function P in terms of x. (iii) Find the profit maximizing level of output of x. (iv) What is the maximum profit, taking rupee as a unit of money ?	
Ans.	Here $C = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$ i.e., $p = 100 - x$ (i) R, the revenue function is $R = px = (100 - x)x = 100\,x - x^2$	½
	(ii) Profit function $P(x) = R(x) - C(x)$ $= (100\,x - x^2) - \left(\frac{x^3}{3} - 7x^2 + 111x + 50\right)$ $= -\frac{x^3}{3} + 6x^2 - 11x - 50$	1
	(iii) $\frac{dP}{dx} = -x^2 + 12x - 11$	½
	For P to be maximum, $\frac{dP}{dx} = 0 \Rightarrow x = 1, 11$	1
	$\frac{d^2P}{dx^2} = -2x + 12 > 0$ at $x = 1$ and < 0 at $x = 11$ Thus P is maximum when $x = 11$	1
	Hence, the profit maximising level of output is 11 units (iv) Maximum profit = $[P(x)]_{x=11}$ $= -\frac{(11)^3}{3} + 6(11)^2 - 11(11) - 50$ $= 111.33\ or\ 111$	1
33(a).	A company establishes a sinking fund to provide for the payment of ₹ 1,00,000 debt, maturing in 4 years. Contributions to the fund are to be made at the end of year. Find the amount of each annual deposit if interest is 18% per annum. [Use $(1{\cdot}18)^4 = 1{\cdot}9388$]	
Ans.	Let each annual deposit to the sinking fund be ₹ R $\therefore 100000 = R\left[\frac{(1+0.18)^4 - 1}{0.18}\right]$ $= R\left[\frac{(1.18)^4 - 1}{0.18}\right]$	2

	$= R\left[\frac{0.4388}{0.18}\right] = R(5.2186)$ $\Rightarrow$ R = $\frac{100000}{5.2186}$ = ₹ $\mathbf{19162.22}$	1½ 1½
	OR	
33(b).	A firm bought a machinery for ₹ 7,40,000 on 1st April, 2020 and ₹ 60,000 is spent on its installation. Its useful life is estimated to be of 5 years. Its scrap value at the end of 5 years is estimated to be ₹ 40,000. Find the amount of annual depreciation and the rate of depreciation.	
Ans.	$\boldsymbol{C}$ = ₹ $\mathbf{7,40,000}$ + ₹ $\mathbf{60,000}$ = ₹ $\mathbf{8,00,000}$ And $\boldsymbol{S}$ = ₹ $\mathbf{40,000}$ ∴ Annual depreciation = $\frac{\boldsymbol{C-S}}{\boldsymbol{n}}$ = ₹ $\frac{\mathbf{7,60,000}}{\mathbf{5}}$ = ₹ $\mathbf{1,52,000}$ Rate of depreciation = $\frac{\boldsymbol{D}}{\boldsymbol{C-S}} \times \mathbf{100}$ $= \frac{\mathbf{1,52,000}}{\mathbf{8,00,000-40,000}} \times \mathbf{100}$ $= \mathbf{20\%}$	1 1 1 1 1
34.	A person takes a housing loan worth ₹ 10,00,000 at an interest rate of 6% p.a compounded monthly. He decided to repay the loan by equal monthly instalments in 15 years. Calculate the EMI, using (i) flat rate method, (ii) reducing balance method. [Given : $(1{\cdot}005)^{-180} = 0{\cdot}4074824$]	
Ans.	Here, P = ₹ $10,00,000, i = \frac{6}{12\times100} = 0.005$ $n = 15$ years $= 180$ months (i) Using flat rate method EMI $= P\left(i + \frac{1}{n}\right)$ $= 10,00,000\left(0.005 + \frac{1}{180}\right)$ = ₹ 10555.55 (ii) Using reducing balancing method EMI $= \frac{Pi}{1-(1+i)^{-n}}$ $= \frac{10,00,000\times0.005}{1-(1.005)^{-180}}$	1 1+½ 1

$= \frac{5000}{1-0.4704824} = ₹\ 8438.57$ **1½**

35. A library has to accommodate two different types of books on a shelf. The books are each 6 cm and 4 cm thick and each weighs 1 kg and $1\frac{1}{2}$ kg respectively. The shelf is 96 cm long and can support a weight of atmost 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books ? Formulate it as an L.P.P and so solve it graphically.

Ans.

Types of boxes	Thickness (in cm)	Weight (in kg)
Type 1	6	1
Type 2	4	$1\frac{1}{2}$
Max Availability	96 cm	21 kg

Let the two types of boxes be x and y respectively

Let Z denote the maximum number of books that can be accommodated in the shelf

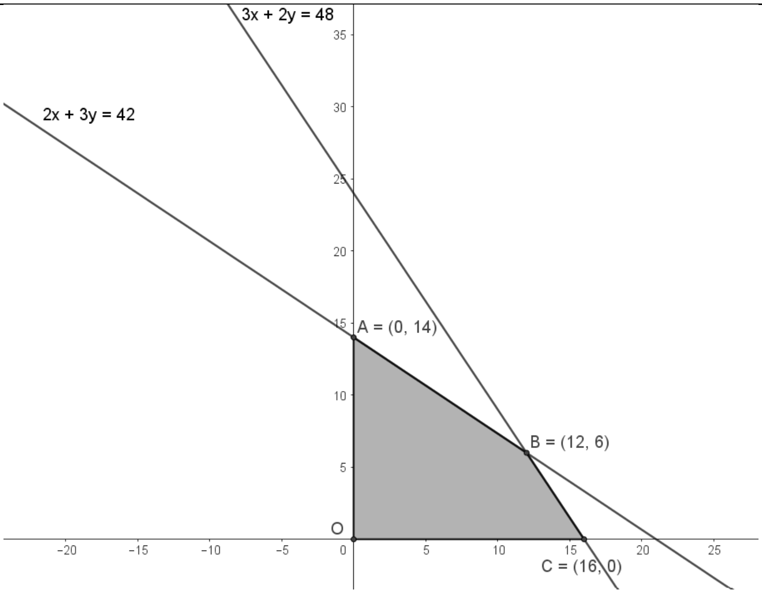
LPP is

$\therefore Z = x + y$ **1**

Subject to constraints

$$\left.\begin{array}{l} 6x + 4y \leq 96 \text{ or } 3x + 2y \leq 48 \\ x + \frac{3}{2}y \leq 21 \text{ or } 2x + 3y \leq 42 \\ \qquad x, y \geq 0 \end{array}\right\}$$

1½

	 Here, $(Z)_A = 14, (Z)_B = 18, (Z)_C = 16$ So, Z is maximum at B Hence, the shelf should be filled with 12 books of type 1 and 6 books of type 2	**1½ For graph with correct region** **1**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	

36.

Case Study – 1

10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students, and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and the second group is four times that of the third group. Assume that x, y and z denote the number of students in first, second and third group respectively.

Based on the above information, answer the following questions :

(a) Write the system of linear equations that can be formulated from the above described situation. *1*

(b) Write the coefficient matrix, say A. *1*

(c) (i) Write the matrix of cofactors of every element of matrix A. *2*

OR

(c) (ii) Determine the number of students of each group. *2*

Ans.

(a) $x + y + z = 10, 2x + y = 13, x + y = 4z$ **1**

(b) coefficient matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$ **1**

(c) (i) Cofactor matrix of $A = \begin{bmatrix} -4 & 8 & 1 \\ 5 & -5 & 0 \\ -1 & 2 & -1 \end{bmatrix}$ **2**

OR

(ii) Adj (A) $= \begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ and $|A| = 5 \neq 0$

$$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$$ **1**

$$\text{Thus, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

∴ 5 students in first group, 3 students in second group and 2 students in third group **1**

37.	**Case Study – 2** A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation $y = 40,00,000 - (x - 2000)^2$, where y is the total revenue and x is the number of units sold. Based on the above information, answer the following questions : (a) Find what number of units sold maximizes total revenue. *2* (b) What is the amount of this maximum revenue ? *1* (c) What would be the total revenue if 2500 units were sold ? *1*	
Ans.	(a) $y = 40,00,000 - (x - 2000)^2$ Gives $\frac{dy}{dx} = -2(x - 2000)$ So, $\frac{dy}{dx} = 0$ when $x = 2000$ $\frac{d^2y}{dx^2} = -2 < 0$ Hence y is max at $x = 2000$ (b) Max Revenue $= 40,00,000 - (2000 - 2000)^2 = 40,00,000$ (c) Total Revenue $= 40,00,000 - (2500 - 2000)^2$ $= 37,50,000$	**1½** **½** **1** **1**

38.

Case Study – 3

The following data shows the percentage of rural, urban and sub-urban Indians who have high speed internet connection at home.

Year	Rural	Urban	Sub-urban
2016	3	9	9
2017	6	18	17
2018	9	21	23
2019	16	29	29
2020	24	38	40

Based on the above information, answer the following questions :

(a) Derive straight-line trend by the method of least squares for the rural students. *2*

OR

(a) Derive straight-line trend by the method of least squares for the urban Indians. *2*

(b) What is the forecast for the year 2021 for urban group using trend equation ? *1*

(c) What is the forecast for the year 2021 for rural group using trend equation ? *1*

Ans. (a)

y	x	x^2	xy
3	-2	4	-6
6	-1	1	-6
9	0	0	0
16	1	1	16
24	2	4	48
$\sum y = 58$		$\sum x^2 = 10$	$\sum xy = 52$

1½

Trend value is $y = \frac{\sum y}{5} + \frac{\sum xy}{\sum x^2} x$ **½**

$$y = 11.6 + 5.2\,x$$

OR

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<tr><th>y</th><th>x</th><th>x^2</th><th>xy</th></tr>
<tr><td>9</td><td>-2</td><td>4</td><td>-18</td></tr>
<tr><td>18</td><td>-1</td><td>1</td><td>-18</td></tr>
<tr><td>21</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>29</td><td>1</td><td>1</td><td>29</td></tr>
<tr><td>38</td><td>2</td><td>4</td><td>76</td></tr>
<tr><td>$\sum y = 115$</td><td></td><td>$\sum x^2 = 10$</td><td>$\sum xy = 69$</td></tr>
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Trend value is $y = \frac{115}{5} + \frac{69}{10}x$
$y = 23 + 6.9\,x$
(b) For $x = 3$, we have
$y = 23 + 6.9\,(3) = 43.7$
(c) For $x = 3$, we have
$y = 11.6 + 5.2\,(3) = 27.2$
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½
1
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