MARKING SCHEME
Strictly Confidential
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Senior Secondary School Supplementary Examination, July 2024
APPLIED MATHEMATICS (241) PAPER CODE – 465/S

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past :- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. NO.	EXPECTED ANSWER / VALUE POINT	MARKS
SECTION A		
Questions no. 1 to 18 are multiple choice questions (MCQs) of 1 mark each.		
Q1	In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price ? (A) 1 : 6 (B) 6 : 1 (C) 3 : 2 (D) 2 : 3	
Ans	(A) 1 : 6	1
Q2	In a 100 m race, A can beat B by 25 m and B can beat C by 4 m. By how much can A beat C in the same race ? (A) 32 m (B) 28 m (C) 24 m (D) 20 m	
Ans	(B) 28 m	1
Q3	If $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $(A^2 - 6A)$ is equal to : (A) $3I$ (B) $-5I$ (C) $5I$ (D) $-3I$	
Ans	(B) $-5I$	1
Q4	If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is : (A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 2	
Ans	(B) $\frac{1}{2}$	1

Q5	$\int 2^{2x} \cdot 3^x \, dx$ is equal to : (A) $\frac{12^x}{\log 12} + C$ (B) $\frac{2^{2x} \cdot 3^x}{\log 2 \cdot \log 3} + C$ (C) $\frac{4.6^x}{\log 6} + C$ (D) $(12)^x \cdot \log 12 + C$	
Ans	(A) $\frac{12^x}{\log 12} + C$	1
Q6	If $x + y = 8$, then the maximum value of (xy) is : (A) 12 (B) 16 (C) 20 (D) 24	
Ans	(B) 16	1
Q7	The demand curve for a monopolist is given by $x = 100 - 4p$. The value of x for which MR = 0, is : (A) 25 (B) 30 (C) 45 (D) 50	
Ans	(D) 50	1
Q8	A random variable X takes the values -1, 0, 1. If its mean is 0.6 and $P(X = 0) = 0.2$, then $P(X = 1)$ is : (A) 0.7 (B) 0.5 (C) 0.4 (D) 0.3	
Ans	(A) 0.7	1

Q9	One hundred identical coins each with probability p showing up heads are tossed once. If $0 < p < 1$ and the probability of heads on 50 coins is equal to that of heads showing on 51 coins, then the value of p is : (A) $\frac{1}{2}$ (B) $\frac{49}{101}$ (C) $\frac{50}{101}$ (D) $\frac{51}{101}$	
Ans	(D) $\frac{51}{101}$	1
Q10	The probability that a bomb dropped from a plane strikes the target is $\frac{4}{5}$. What is the probability that out of 6 bombs dropped, exactly 2 bombs strike the target ? (A) $2\left(\frac{4}{5}\right)^5$ (B) $1-2\left(\frac{4}{5}\right)^5$ (C) $\frac{48}{5^5}$ (D) $\frac{64}{5^6}$	
Ans	(C) $\frac{48}{5^5}$	1
Q11	A specific characteristic of a sample is known as a : (A) population (B) parameter (C) statistic (D) variance	
Ans	(C) statistic	1
Q12	The test statistic for a one sample t-test, denoted by t, is defined as : (A) $t = \frac{\overline{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)}$ (B) $t = \frac{\overline{x} - \mu}{\left(\frac{S}{n}\right)}$ (C) $t = \frac{\overline{x} - \mu}{\left(\frac{S^2}{n}\right)}$ (D) $t = \frac{\overline{x} - \mu}{\left(\frac{S}{n^2}\right)}$ where μ is the population mean and $\overline{x}$ is the sample mean.	
Ans	(A) $t = \frac{\overline{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)}$	1

Q13	If for a data, n = 6, $\Sigma y = 84$, $\Sigma xy = 108$, $\Sigma x^2 = 70$ and $\Sigma x = 0$, then the equation of the straight line trend is : (A) $y_c = 14 + 1.54x$ (B) $y_c = 1.54 + 14x$ (C) $y_c = 14 + 3.08x$ (D) $y_c = 3.08 + 14x$	
Ans	(A) $y_c = 14 + 1.54x$	1
Q14	At what rate of interest will the present value of a perpetuity of ₹ 500 payable at the end of each quarter be ₹ 40,000 ? (A) 1.25% p.a. (B) 2.5% p.a. (C) 5% p.a. (D) 6% p.a.	
Ans	(C) 5% p.a.	1
Q15	If nominal rate is r% compounded k times in a year, then the effective rate of interest r_e is given by : (A) $r_e = \left(1 - \frac{r}{100k}\right)^k - 1$ (B) $r_e = \left(1 + \frac{r}{100k}\right)^k + 1$ (C) $r_e = \left(1 - \frac{r}{100k}\right)^k + 1$ (D) $r_e = \left(1 + \frac{r}{100k}\right)^k - 1$	
Ans	(D) $r_e = \left(1 + \frac{r}{100k}\right)^k - 1$	1
Q16	If the annual depreciation of an asset is ₹ 40,000 and its scrap value after useful life of 15 years is ₹ 50,000, then the original cost of the asset is : (A) ₹ 7,60,000 (B) ₹ 7,20,000 (C) ₹ 6,50,000 (D) ₹ 6,30,000	
Ans	(C) ₹ 6,50,000	1

Q17	The number of solutions of an L.P.P. to minimize $z = 3x + 2y$ under the constraints $x + y \geq 8$, $3x + 5y \leq 15$ and $x, y \geq 0$, is : (A) 2 (B) 5 (C) infinitely many (D) zero	
Ans	(D) zero	1
Q18	An investment's starting value is ₹ 10,000 and it grows to ₹ 60,000 in 4 years. The CAGR is : (Given : $6^{1/4} = 1.56508$) (A) 1.56% (B) 5.65% (C) 15.65% (D) 56.50%	
Ans	(D) 56.50%	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19	*Assertion (A) :* The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is 3. *Reason (R) :* The highest power of the highest order derivative involved in a differential equation, when it is written as a polynomial in derivatives, is called its degree.	
Ans	(D) Assertion (A) is false, but Reason (R) is true. *(Note: Option (A) may be considered correct since students are not familiar with trigonometric functions.)*	1

Q20	*Assertion (A) :* Minor of element a_{13} in the matrix $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$ is $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$. *Reason (R) :* Minor of an element a_{ij} of a matrix is the determinant obtained by deleting its j^{th} row and i^{th} column in which the element lies.	
Ans	(C) Assertion (A) is true, but Reason (R) is false.	1
SECTION B Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.		
Q21(a)	Evaluate (137 + 995) (mod 12).	
Ans	$(137+995)(\text{mod}\,12)$ $\equiv 137(\text{mod}\,12)+995(\text{mod}\,12)$ $\equiv 5(\text{mod}\,12)+11(\text{mod}\,12)$ $\equiv (5+11)(\text{mod}\,12)$ $\equiv 16(\text{mod}\,12)$ $\equiv 4(\text{mod}\,12)$ Hence $(137+995)(\text{mod}\,12)=4$	1 1
OR		
Q21(b)	Find the unit's digit of 12^{12}.	
Ans	Let us evaluate $12^{12}(\text{mod}\,10)$ We know that $12 \equiv 2(\text{mod}\,10)$ $\Rightarrow 12^4 \equiv 2^4(\text{mod}\,10)$ $\equiv 16(\text{mod}\,10)$ $\equiv 6(\text{mod}\,10)$ $\Rightarrow 12^{12} = (12^4)^3 \equiv 6^3(\text{mod}\,10)$ $\equiv 216(\text{mod}\,10)$ $\equiv 6\ (\text{mod}\ 10)$ Hence, the unit's digit of 12^{12} is 6.	1 1
Q22	If $y = \left(x + \sqrt{x^2+1}\right)^p$, then prove that $(x^2+1)y_2 + xy_1 - p^2y = 0$; where $y_1 = \dfrac{dy}{dx}$ and $y_2 = \dfrac{d^2y}{dx^2}$.	

Ans	$y=\left(x+\sqrt{x^2+1}\right)^p$ $\Rightarrow y_1=p\left(x+\sqrt{x^2+1}\right)^{p-1}\left[1+\frac{2x}{2\sqrt{x^2+1}}\right]$ $=\frac{p}{\sqrt{x^2+1}}\left(x+\sqrt{x^2+1}\right)^p$ $=\frac{py}{\sqrt{x^2+1}}$ $\Rightarrow\left(x^2+1\right)y_1^2=p^2y^2$ Differentiating both sides, we get $2xy_1^2+2\left(x^2+1\right)y_1y_2=2p^2yy_1$ $\Rightarrow xy_1+\left(x^2+1\right)y_2=p^2y$ $\Rightarrow\left(x^2+1\right)y_2+xy_1-p^2y=0$	1 ½ ½
Q23(a)	If X is a normal variate with mean (μ) = 70 and standard deviation (σ) = 5, then find P(X > 75). (Given : P(0 < Z < 1) = 0.3413)	
Ans	For $X=75,\ Z=\frac{75-70}{5}\Rightarrow Z=1$ $P(X>75)=P(Z>1)$ $=0.5-P(0<Z<1)$ $=0.5-0.3413$ $=0.1587$	½ 1 ½
OR		
Q23(b)	If X is a Poisson variate such that P(X = 0) = P(X = 1) = α, then show that $\alpha = e^{-1}$.	
Ans	Let X be a poisson variate with parameter λ. Then $P(X=r)=\frac{e^{-\lambda}\lambda^r}{r!}$ Here $P(X=0)=e^{-\lambda}$ and $P(X=1)=\lambda e^{-\lambda}$ $\therefore P(X=0)=P(X=1)=\alpha \Rightarrow e^{-\lambda}=\lambda e^{-\lambda}=\alpha$ $\Rightarrow\lambda=1$ $\Rightarrow\alpha=\frac{1}{e}$ i.e. $\alpha=e^{-1}$	1 ½ ½

Q24

Find the trend values by taking five yearly moving averages for the following data :

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020
Annual Production (Million tons)	16	14	20	18	22	17	19	21	20

Ans

Year	**Production**	**Five yearly moving total**	**Five yearly moving averages**
2012	16	-	-
2013	14	-	-
2014	20	16+14+20+18+22=90	90/5=18
2015	18	14+20+18+22+17=91	91/5=18.2
2016	22	20+18+22+17+19=96	96/5=19.2
2017	17	18+22+17+19+21=97	97/5=19.4
2018	19	22+17+19+21+20=99	99/5=19.8
2019	21	-	-
2020	20	-	-

1 mark each for the 3rd and 4th column

Hence the trend values are 18, 18.2, 19.2, 19.4, 19.8

Q25

Two tailors A and B earn ₹ 1500 and ₹ 2000 per day, respectively. Tailor A can stitch 6 shirts and 4 pants, while tailor B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Ans

Let us assume that tailor A works for x days and tailor B works for y days to complete the job.

Let Z denote the total labour cost.

The LPP for the given problem is:

Minimize $Z = 1500x + 2000y$ ½

subject to constraints,

$6x + 10y \geq 60$ or $3x + 5y \geq 30$ ½

$4x + 4y \geq 32$ or $x + y \geq 8$ ½

$x \geq 0, y \geq 0$ ½

	SECTION C Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.	
Q26(a)	Solve $3x + 8 > 2$, when (i) x is an integer. (ii) x is a natural number. (iii) x is a whole number.	
Ans	$3x+8>2 \Rightarrow x>-2$ (i) solution set $=\{-1,0,1,2,3,...\}$ (ii) solution set $=\{1,2,3,...\}$ (iii) solution set $=\{0,1,2,3,...\}$	 1 1 1
	OR	
Q26(b)	A man goes 12 km downstream and comes back to the starting point by swimming non-stop in 3 hours. If the speed of the stream is 3 km/h, find the speed with which the man can swim in still water.	
Ans	Let speed of man in the still water be x km/h. Speed of stream is 3 km/h Speed downstream = (x+3) km/h and speed upstream is ($x-3$) km/h According to the given question, $\frac{12}{x-3}+\frac{12}{x+3}=3$ $\Rightarrow x^2-8x-9=0$ $\Rightarrow (x+1)(x-9)=0$ $\Rightarrow x=-1, x=9$ $\Rightarrow x=9 \quad (x=-1 \text{ is rejected})$ Thus, the man can swim in still water at 9 km/h.	 1 1 1
Q27(a)	Evaluate : $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$	
Ans	$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ $=1(yz^2-y^2z)-1(xz^2-x^2z)+1(xy^2-x^2y)$ $=yz^2-y^2z-xz^2+x^2z+xy^2-x^2y$ *(Note: Alternatively, if a student has used the properties of determinants, then the final answer will be* $(x-y)(y-z)(z-x)$. *Appropriate marks may be given.)*	 2 1

Q27(b)	Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.	
Ans	Here $\|A\| = 1(3-0) - 2(-1-0) - 2(2-0)$ $= 3+2-4 = 1$ $A_{11} = 3$, $A_{12} = 1$, $A_{13} = 2$ $A_{21} = 2$, $A_{22} = 1$, $A_{23} = 2$ $A_{31} = 6$, $A_{32} = 2$, $A_{33} = 5$ cofactor matrix $= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$ $\Rightarrow adjA = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{\|A\|} adjA = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$	1 1 1
Q28	Ten students are selected at random from a college and their heights (in cm) are found to be 100, 104, 108, 110, 118, 120, 122, 124, 126 and 128. In the light of the data, discuss the conclusion that the mean height of the students of the college is 110 cm. [Given : t_9 (0.05) = 2.262]	
Ans	Null hypothesis H_0 : There is no significant difference between the sample mean and population mean. Alternate hypothesis H_1 : The sample mean is not the same as population mean. Let the sample statistic t is given by $t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$ For the given data : $n = 10$, $\sum_{i=1}^{10} x_i = 1160$, $\bar{X} = 116$ and $\sum_{i=1}^{10} (x_i - \bar{X})^2 = 864$ $\Rightarrow s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{10} (x_i - \bar{X})^2} = \frac{\sqrt{864}}{3}$ Thus, $t = \frac{116 - 110}{\frac{\sqrt{864}}{3}} \times \sqrt{10} = \frac{\sqrt{15}}{2} \approx 2$ Since $\|t\| < 2.262$, the null hypothesis is accepted. i.e. mean height of the students of the college is 110 cm.	½ 1 1 ½

Q29	Given below is the data of workers welfare expenses (in lakh ₹) in steel industries during 2016 – 2020 : | *Year* | 2016 | 2017 | 2018 | 2019 | 2020 | | *Workers welfare expenses (in lakh ₹)* | 160 | 185 | 220 | 300 | 510 | Find the best fitted trend line by the method of least squares and tabulate the trend values.	

Year (x_i)	Index number (Y)	$X = x_i - A$ $= x_i - 2018$	X^2	XY
2016	160	-2	4	-320
2017	185	-1	1	-185
2018	220	0	0	0
2019	300	1	1	300
2020	510	2	4	1020
$n=5$	$\sum Y = 1375$	$\sum X = 0$	$\sum X^2 = 10$	$\sum XY = 815$

Ans — 1 mark for the correct table

Now, $a = \dfrac{\sum Y}{n} = \dfrac{1375}{5} = 275$ and $b = \dfrac{\sum XY}{\sum X^2} = \dfrac{815}{10} = 81.5$

Thus the required equation of the best fitted trend line is

$y = a + bx \Rightarrow y = 275 + 81.5x$ — 1

Trend values are :

$2016 \rightarrow 275 + 81.5(-2) = 112$

$2017 \rightarrow 275 + 81.5(-1) = 193.5$

$2018 \rightarrow 275 + 81.5(0) = 275$ — 1

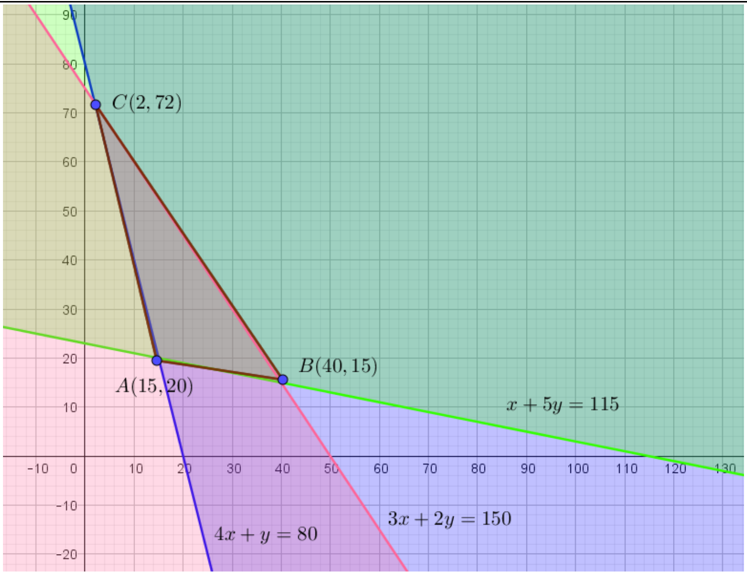
$2019 \rightarrow 275 + 81.5(1) = 356.5$

$2020 \rightarrow 275 + 81.5(2) = 438$

Q30 A machine costing ₹ 2,00,000 has effective life of 7 years and its scrap value is ₹ 30,000. What amount should the company put into a sinking fund earning 5% p.a. so that it can replace the machine after its useful life ? Assume that a new machine will cost ₹ 3,00,000 after 7 years.

[Given : $(1.05)^7 = 1.407$]

Ans	Here, $A = ₹(3,00,000 - 30,000) = ₹\,2,70,000$ We know, $A = R\left[\frac{(1+i)^n - 1}{i}\right]$ $\Rightarrow 270000 = R\left[\frac{(1+0.05)^7 - 1}{0.05}\right]$ $\Rightarrow R = \frac{270000 \times 0.05}{(1.05)^7 - 1} = \frac{270000 \times 0.05}{0.407}$ $\Rightarrow R =$ ₹ 33,169.53 Hence, the company should deposit ₹ 33,169.53 at the end of each year for 7 years.	½ 1 1 ½
Q31	The value of a car depreciates by 12.5% every year. By what percent will the value of the car decrease after 3 years and after 5 years ?	
Ans	*Remark: Full marks may be awarded to the student who has attempted the question as reducing balance method is not explicitly mentioned in the curriculum.*	3
	SECTION D Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.	
Q32	A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of Vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of Vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of Vitamin A ? What is the minimum amount of Vitamin A ? Formulate the above problem as an L.P.P. and solve it graphically.	
Ans	Let x packets of food P and y packets of food Q be mixed. The LPP is: Minimize $Z = 6x + 3y$ subject to constraints $12x + 3y \geq 240$ or $4x + y \geq 80$ $4x + 20y \geq 460$ or $x + 5y \geq 115$ $6x + 4y \leq 300$ or $3x + 2y \leq 150$ $x \geq 0, y \geq 0$	½ 1½



2 marks for the correct graph.

ABC forms the feasible bounded region, where $A(15, 20)$, $B(40, 15)$ and $C(2, 72)$.
Now, $Z_A = 150, Z_B = 285, Z_C = 228$
Z is minimum at $A(15, 20)$.
$Z_{\min} = 150$, when 15 packets of food P and 20 packets of food Q are mixed. — 1

Q33(a) A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made a square and the other, a circle. What would be the lengths of the two pieces, so that the combined area of the square and the circle is minimum ?

Ans Let the length of the one piece bent into shape of a square be x m then the length of the other piece bent into the shape of a circle is $(36 - x)$ m.
Let side of square be 'a' and radius of circle be 'r'.
Then,

$$4a = x, 2\pi r = 36 - x \Rightarrow a = \frac{x}{4}, r = \frac{36 - x}{2\pi}$$ — 1

Combined area $A = a^2 + \pi r^2$

$$\Rightarrow A = \frac{x^2}{16} + \frac{1}{4\pi}(36 - x)^2$$ — 1

$$\frac{dA}{dx} = \frac{x}{8} - \frac{1}{2\pi}(36 - x)$$ — 1

For maxima/minima, put $\frac{dA}{dx} = 0$

$$\Rightarrow \frac{x}{8} = \frac{1}{2\pi}(36 - x) \Rightarrow x = \frac{144}{\pi + 4} m$$ — 1

Also, $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$ at $x = \frac{144}{\pi + 4}$, so A is minimum. — ½

Lengths of two pieces are $\frac{144}{\pi + 4} m$ and $\frac{36\pi}{\pi + 4} m$ resp[c]tively. — ½

OR

Q33(b)	Find : $$\int \frac{x^3}{x^4 + 3x^2 + 2} dx$$	
Ans	$I = \int \frac{x^3}{x^4 + 3x^2 + 2} dx$ Put $x^2 = t \Rightarrow x\,dx = \frac{dt}{2}$ $\therefore I = \frac{1}{2}\int \frac{t}{t^2 + 3t + 2} dt$ $= \frac{1}{2}\int \frac{t}{(t+1)(t+2)} dt$ $= \frac{1}{2}\int \left(-\frac{1}{t+1} + \frac{2}{t+2}\right) dt$ $= \frac{1}{2}\left[-\log\|t+1\| + 2\log\|t+2\|\right] + C$ $= \frac{1}{2}\left[-\log\|x^2+1\| + 2\log\|x^2+2\|\right] + C$ or $\frac{1}{2}\left[\log\frac{(x^2+2)^2}{x^2+1}\right] + C$	 ½ ½ 1 1½ 1 ½
Q34(a)	An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.	
Ans	Let A be the event of obtaining two sixes in the first five throws of a die and B be the event of obtaining a six in the sixth throw of a die. Then, required probability = $P(AB) = P(A).P(B)$ Let p denotes the probabilty of success i.e. getting a six in a single throw of a die. Then, $p = \frac{1}{6}, q = 1 - p = \frac{5}{6}$ $\therefore P(A) = {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{625}{3888}$ Also, $P(B) = \frac{1}{6}$ Hence , required probability = $P(A).P(B)$ $= \frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$	 1 2 1 1

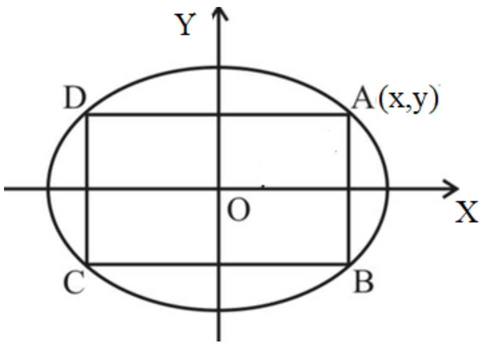
	OR	
Q34(b)	An aptitude test for selecting officers in a bank is conducted on 1000 candidates. The mean score obtained is 42 and the standard deviation of score is 24. Assuming normal distribution for the scores, find : (i) the number of candidates whose scores exceed 60; (ii) the number of candidates whose scores lie between 30 and 60. [Given : $P(0 \le Z \le 0.75) = 0.2734$; $P(0 \le Z \le 0.5) = 0.1915$]	
Ans	Here, $Z = \frac{X-42}{24}$ (i) when $X = 60$, we obtain $Z = \frac{60-42}{24} = \frac{3}{4} = 0.75$ $\therefore P(X > 60) = P(Z > 0.75)$ $= 0.5 - P(0 \le Z \le 0.75)$ $= 0.5 - 0.2734$ $= 0.2266$ Thus, 226 (or 227) students scored more than 60 marks. (ii) when $X = 30$, we obtain $Z = \frac{30-42}{24} = -0.5$ when $X = 60, Z = 0.75$ $\therefore P(30 < X < 60) = P(-0.5 < Z < 0.75)$ $= P(-0.5 < Z < 0) + P(0 \le Z \le 0.75)$ $= P(0 \le Z \le 0.5) + P(0 \le Z \le 0.75)$ $= 0.1915 + 0.2734$ $= 0.4649$ Thus, 464 (or 465) students scored marks between 30 and 60.	½ ½ ½ ½ ½ 1 ½ ½ ½
Q35	Mahesh purchased a house from a company for ₹ 70,00,000 and made a down payment of ₹ 15,00,000. He repays the balance in 25 years by monthly instalments at 9% p.a. compounded monthly. (i) What is the amount of monthly payment ? (ii) What is the total interest payment ? [Given : $(1.0075)^{-300} = 0.1062878338$]	
Ans	$\left.\begin{array}{l}\text{We have, } P = ₹(70,00,000 - 15,00,000) = ₹55,00,000 \\ n = 25 \times 12 = 300 \text{ and } i = \frac{9}{1200} = 0.0075\end{array}\right\}$	1

	(i) Let E be the monthly installment. Then, $E=\dfrac{Pi}{1-(1+i)^{-n}}$ $=\dfrac{5500000\times 0.0075}{1-0.1062878338}$ $=\dfrac{41250}{0.8937121662}=46155.80$ ∴ The monthly installment is ₹ 46,155.80 (ii) Total interest $=nE-P$ $=300\times 46155.80-5500000$ $=13846740-5500000$ = ₹ 83,46,740 *(Note: The student is allowed to use any approximate value of 0.1062878338)*	 1 1 1 1

SECTION E

Questions no. 36 to 38 are case study based questions carrying 4 marks each.

Q36	**Case Study – 1** Rohini wants to give a rectangular plot of land for a school in her village. When she was asked to mention the dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area does not alter, but if its length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 sq m. Based on the above information, answer the following questions : (i) Assuming x m and y m as the length and breadth of the plot respectively, write the system of linear equations in x and y. *1* (ii) Write the system of linear equations obtained in (i) in the matrix equation AX = B. *1* (iii) (a) Determine A^{-1}. *2* **OR** (b) Find the area of the plot. *2*	
Ans(i)	$(x-50)(y+50)=xy \Rightarrow x-y=50$ and $(x-10)(y-20)+5300=xy \Rightarrow 2x+y=550$	½ ½
Ans(ii)	$\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}50\\ 550\end{bmatrix}$	1

	where, $A=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}50\\ 550\end{bmatrix}$	
Ans (iii) (a)	$\lvert A\rvert=3$ and $adjA=\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}$ $\therefore\ A^{-1}=\frac{1}{3}\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}$	1 1
	OR	
Ans (iii)(b)	$X=\begin{bmatrix}x\\ y\end{bmatrix}=A^{-1}B=\frac{1}{3}\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}\begin{bmatrix}50\\ 550\end{bmatrix}=\frac{1}{3}\begin{bmatrix}600\\ 450\end{bmatrix}=\begin{bmatrix}200\\ 150\end{bmatrix}$ $\therefore \text{Area}=xy=200\times150=30,000\ \text{sq m}$	1½ ½
Q37	**Case Study – 2** Read the following passage and answer the questions given below : "In an elliptical sports field, the authority wants to design a rectangular soccer field with the maximum possible area. The sports field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$." (i) If the length and breadth of the rectangular soccer field be 2x and 2y respectively, then find the area function A(x) in terms of x. *1* (ii) Find the critical point(s) of the area function A(x). *1* (iii) (a) Using first derivative test, find the length 2x and breadth 2y of the soccer field (in terms of a and b) that maximize the area. *2* **OR** (b) Using second derivative test, find the length 2x and breadth 2y of the soccer field (in terms of a and b) that maximize the area. *2*	
Ans(i)	 Area of soccer field = $(2x)(2y)=4xy$ $\Rightarrow A(x)=4x\times\frac{b}{a}\sqrt{a^2-x^2}=\frac{4bx}{a}\sqrt{a^2-x^2}$	 ½ ½

Ans(ii)	$\frac{dA}{dx}=\frac{4b}{a}\left(\sqrt{a^2-x^2}+\frac{x(-2x)}{2\sqrt{a^2-x^2}}\right)$ or $\frac{dA}{dx}=\frac{4b}{a}\left(\frac{a^2-2x^2}{\sqrt{a^2-x^2}}\right)$ $\frac{dA}{dx}=0\Rightarrow x=\pm\frac{a}{\sqrt{2}}$ $\therefore$ Critical point is $x=\frac{a}{\sqrt{2}}$	½ ½
Ans (iii)(a)	$\frac{dA}{dx}>0$ for values of x close to $\frac{a}{\sqrt{2}}$ and to the left of $\frac{a}{\sqrt{2}}$ and $\frac{dA}{dx}<0$ for values of x close to $\frac{a}{\sqrt{2}}$ and to the right of $\frac{a}{\sqrt{2}}$ $\therefore A$ is maximum at $x=\frac{a}{\sqrt{2}}$. Length $=2x=\sqrt{2}\,a$ and breadth $=2y=\sqrt{2}\,b$	1 1
OR		
Ans (iii)(b)	$\frac{d^2A}{dx^2}=\frac{4b}{a}\left[\frac{\left(\sqrt{a^2-x^2}\right)(-4x)-\left(a^2-2x^2\right)\left(\frac{-2x}{2\sqrt{a^2-x^2}}\right)}{\left(a^2-x^2\right)}\right]<0$ at $x=\frac{a}{\sqrt{2}}$ $\therefore A$ is maximum at $x=\frac{a}{\sqrt{2}}$. Length $=2x=\sqrt{2}\,a$ and breadth $=2y=\sqrt{2}\,b$	1 1
Q38	For providing water to the families of a colony, a large water tank with two inlet pipes A and B and an outlet pipe C, is installed. Pipes A and B can fill the tank in 10 hours and 12 hours respectively; whereas pipe C can empty the tank in 15 hours. Based on the above information, answer the following questions : (i) If both pipes A and B are opened together, then find the time in which the tank will be filled completely. *1* (ii) If both pipes A and C are opened together, then find the time in which the tank will be filled completely. *1* (iii) (a) If all the three pipes A, B and C are opened together, then find the time in which the tank will be filled completely. *2* **OR** (b) Pipes A and B are opened together for some time and then pipe B is turned off after some time. If the tank is completely filled in 6 hours, then after how many hours is pipe B turned off ? *2*	
Ans(i)	Part of the tank filled by pipes A and B in one hour $=\frac{1}{10}+\frac{1}{12}=\frac{11}{60}$	½

	So, two pipes will fill the tank in $\frac{60}{11}$ hours.	½
Ans(ii)	Part of the tank filled by pipes A and C in one hour $=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$ So, two pipes will fill the tank in 30 hours.	½ ½
Ans (iii)(a)	Part of the tank filled by three pipes A, B and C in one hour $=\frac{1}{10}+\frac{1}{12}-\frac{1}{15}=\frac{7}{60}$ So, these three pipes will fill the tank in $\frac{60}{7}$ hours.	1½ ½
	OR	
Ans (iii)(b)	Let the pipe B be turned off after 't' hours. Part of tank filled by A in 6 hours = $\frac{6}{10}$ Part of tank filled by B = $1-\frac{6}{10}=\frac{4}{10}$ $\Rightarrow \frac{t}{12}=\frac{4}{10}$ $\Rightarrow t=4.8$ $\therefore$ Pipe B is turned off after 4.8 hours	1 ½ ½