CBSE-2025-Mathematics
SET-65/5/1

SECTION-A

1. (b)
2. (d)
3. (c)
4. (c)
5. (c)
6. (b)
7. (d)
8. (c)
9. (b)
10. (a)
11. (b)
12. (c)
13. (b)
14. (c)
15. (c)
16. (b)
17. (c)
18. (c)
19. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
20. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

SECTION-B

21. Domain of $\cos^{-1} x$ is $[-1,1]$

$\Rightarrow - \quad 1 \leq x^2 - 4 \leq 1 \Rightarrow 3 \leq x^2 \leq 5$

$\Rightarrow x \quad x[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

22. $\frac{dS}{dt} = 5 \text{ mm}^2/\text{s}, \left(\frac{dV}{dt}\right)_{r=8} = ?$

$$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{8\pi r}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dV}{dt} = \frac{5}{2} r$$

$$\Rightarrow \left(\frac{dV}{dt}\right)_{r=8} = 20 \text{ mm}^3/s$$

23. (a) Let $y = \frac{\sin x}{\sqrt{\cos x}}$

$$\frac{dy}{dx} = \frac{\sqrt{\cos x} \cdot \cos x - \sin x \cdot \left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{3/2}} \text{ or } \frac{1 + \cos^2 x}{2(\cos x)^{3/2}}$$

OR

(b) $y = 5\cos x - 3\sin x$, then $\frac{dy}{dx} = -5 \cdot \sin x - 3 \cdot \cos x$

$\Rightarrow \dfrac{d^2y}{dx^2} = -5 \cdot \cos\, x + 3 \cdot \sin\, x = -y$

$\Rightarrow \dfrac{d^2y}{dx^2} + y = 0$

24. **(a)** Let $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{\imath} + 10\hat{\jmath} + 17\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$

Unit vector $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \dfrac{1}{\sqrt{390}}(\hat{\imath} + 10\hat{\jmath} + 17\hat{k})$

Required vector $= \dfrac{5}{\sqrt{390}}(\hat{\imath} + 10\hat{\jmath} + 17\hat{k})$

OR

(b) $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b} - \vec{c})$, since $\vec{a} \neq 0$

Also, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} \| (\vec{b} - \vec{c})$, since $\vec{a} \neq 0$

Since vectors $\vec{a}$ and $(\vec{b} - \vec{c})$ cannot be $\|$ and $\perp$ simultaneously

Hence $\vec{b} = \vec{c}$

25.

Let P and Q trisect the wire AB.

P divides AB in the ratio $1:2$ then, coordinate of point $P = \left(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

Q divides AB in the ratio $2:1$ then, coordinate of point $Q = \left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$

SECTION-C

26. Since f(x) is a decreasing function $\Rightarrow f'(x) \leq 0$

$\Rightarrow \sqrt{3} \cdot \cos\, x + \sin\, x - 2a \leq 0$

$\Rightarrow 2\left(\dfrac{\sqrt{3}}{2} \cdot \cos\, x + \dfrac{1}{2}\sin\, x\right) - 2a \leq 0$

$\Rightarrow \cos\left(x - \dfrac{\pi}{6}\right) \leq a$

Since, $-1 \leq \cos\left(x - \frac{\pi}{6}\right) \leq 1 \Rightarrow a \geq 1$ i.e. $a \in [1, \infty)$ or $(1, \infty)$

27. **(a)** Let $I = \int \frac{2x}{(x^2+3)(x^2-5)} dx$

Put $x^2 = t \Rightarrow 2x.dx = dt$

$\Rightarrow I = \displaystyle\int \frac{dt}{(t+3)(t-5)}$

$= \displaystyle\int \left(-\frac{1}{8(t+3)} + \frac{1}{8(t-5)}\right) dt$

$= \dfrac{1}{8}[\log\, |t - 5| - \log\, |t + 3|] + c$

$= \dfrac{1}{8}\log\left|\dfrac{x^2 - 5}{x^2 + 3}\right| + c$

OR

(b) $\int_1^4 (|x - 2| + |x - 4|)dx$

$$= \int_1^2 (2-x)dx + \int_2^4 (x-2)dx - \int_1^4 (x-4)dx$$
$$= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-4)^2}{2}\right]_1^4$$
$$= \frac{1}{2} + 2 + \frac{9}{2} = 7$$

28. $\left[x.\sin^2\frac{y}{x} - y\right].dx + x.dy = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x} - \sin^2\dfrac{y}{x}$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = x\dfrac{dv}{dx} + v$

$x\dfrac{dv}{dx} + v = v - \sin^2 v$

$\Rightarrow -\int \text{cosec}^2 v dv = \int \dfrac{dx}{x}$

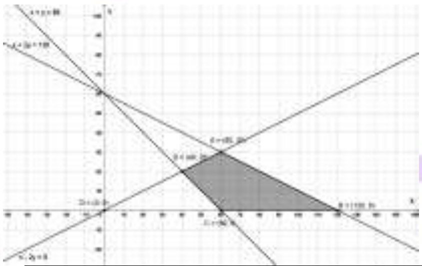
$\Rightarrow \cot v = \log|x| + c$

$\Rightarrow \cot\dfrac{y}{x} = \log|x| + c$

$x = 1, y = \dfrac{\pi}{4} \Rightarrow c = 1$

$\Rightarrow \cot\dfrac{y}{x} = \log|x| + 1$

29.



Corner Points	Value of Z
A(60,30)	600
B(120,0)	600
C(60,0)	300
D(40,20)	400

Since Z is maximum on points A and B

Hence all points lying on segment AB give maximum Z.

30. **(a)** Given $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow |\vec{a} + \vec{b}| = |-\vec{c}|$

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{c}|^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow 9 + 25 + 2\vec{a}\cdot\vec{b} = 49$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta = 15$

$\Rightarrow \cos\theta = \dfrac{1}{2} \therefore \theta = \dfrac{\pi}{3}$

OR

(b) $|\vec{a}| = |\vec{b}| = 1$

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$
$= 1 + 1 - 2|\vec{a}||\vec{b}|\cos\,\theta$
$= 2 - 2\cos\,\theta$
$= 2\left(2\sin^2\dfrac{\theta}{2}\right) = 4\sin^2\dfrac{\theta}{2}$
$\Rightarrow \sin\dfrac{\theta}{2} = \dfrac{1}{2}|\vec{a}-\vec{b}|$

31. (a) Let A be the event of buying colouring book and
B be the event of buying coloured box.
$\mathrm{P(A)} = 0.7, \mathrm{P(B)} = 0.2, \mathrm{P(A/B)} = 0.3$
(i) $\mathrm{P}\left(\dfrac{\mathrm{A}}{\mathrm{B}}\right) = \dfrac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}} \Rightarrow 0.3 = \dfrac{\mathrm{P(A\cap B)}}{0.2}$
$\Rightarrow \mathrm{P(A\cap B)} = 0.06$ or $\dfrac{3}{50}$
(ii) $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$
$=\frac{0.06}{0.7}=\frac{3}{35}$ or 0.086

OR

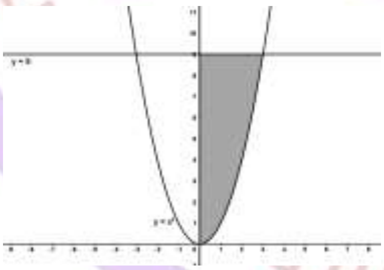
(b) Let **X** be random variable for number of oranges.
$\mathrm{X} = 0,1,2,3$
Let A be the event that orange is drawn.
$P(A) = \dfrac{4}{10} = \dfrac{2}{5}, P(\bar{A}) = 1 - \dfrac{2}{5} = \dfrac{3}{5}$
(i)

X	0	1	2	3
P(X)	$\dfrac{27}{125}$	$\dfrac{54}{125}$	$\dfrac{36}{125}$	$\dfrac{8}{125}$

(ii) $E(X) = \sum p_iX_i = 0\times\frac{27}{125} + 1\times\frac{54}{125} + 2\times\frac{36}{125} + 3\times\frac{8}{125}$
$= \dfrac{150}{125}$ or $\dfrac{6}{5}$

SECTION-D

32.



Required area $= \displaystyle\int_0^9 \sqrt{y}dy$
$= \dfrac{2}{3}\left[y^{3/2}\right]_0^9$
$= 18$

33. Let the numbers of chairs, tables and beds produced be x, y and z respectively.
$x + y + z = 45;\ -x + 0.y + z = 8;\ x - 2y + z = 0$
Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$

$|A| = 1(0+2) - 1(-1-1) + 1(2-0) = 6 \neq 0$

A^{-1} exists

$AX = B \Rightarrow X = A^{-1}B$

$$\operatorname{adj}(A) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$$

So, $\mathrm{x} = 11, \mathrm{y} = 15, \mathrm{z} = 19$

Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.

34. (a) Let $u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}} \cdot \log a \cdot \left(1 - \frac{1}{t^2}\right)$

$$v = \left(t + \frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t + \frac{1}{t}\right)^{a-1} \cdot \left(1 - \frac{1}{t^2}\right)$$

$$\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$$

OR

(b) Let $u = y^x, v = x^y$ and $w = x^x$

$$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \ldots\ldots\ldots\ldots(i)$$

$$u = y^x \Rightarrow \log u = x \cdot \log y \Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \log y$$

$$\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right) = xy^{x-1}\frac{dy}{dx} + y^x \log y$$

$$v = x^y \Rightarrow \log v = y \cdot \log x \Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right) = yx^{y-1} + x^y \log x\frac{dy}{dx}$$

$$w = x^x \Rightarrow \log w = x \cdot \log x \Rightarrow \frac{1}{w} \cdot \frac{dw}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dw}{dx} = x^x \cdot (1 + \log x)$$

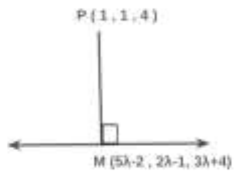
$\therefore$ From (i), we get

$$xy^{x-1} \cdot \frac{dy}{dx} + y^x \cdot \log y + yx^{y-1} + x^y \cdot \log x \cdot \frac{dy}{dx} + x^x \cdot (1 + \log x) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^x \cdot (1 + \log x) + y^x \cdot \log y + yx^{y-1}}{x \cdot y^{x-1} + x^y \cdot \log x}$$

35. (a) Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$

Coordinate of general point on the given line are $M(5\lambda - 2, 2\lambda - 1, 3\lambda + 4)$



Direction Ratios of PM vector are $\langle 5\lambda - 3, 2\lambda - 2, 3\lambda \rangle$

Since, PM $\perp$ $\boldsymbol{l}$

$\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$

$\Rightarrow \lambda = \dfrac{1}{2}$

Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$

OR

(b) Equation of given line be $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ (say)

Coordinate of any general point on the line are $P(3\lambda + 1, 2\lambda - 1, 3\lambda + 4)$.

Let distance of point P from $(-1,-1,2)$ is $2\sqrt{2}$.

$\Rightarrow \sqrt{(3\lambda + 2)^2 + (2\lambda)^2 + (3\lambda + 2)^2} = 2\sqrt{2}$

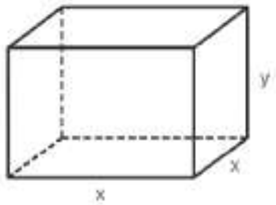
$\Rightarrow 22\lambda^2 + 24\lambda = 0$

$\Rightarrow \lambda = 0$ or $\lambda = -\dfrac{12}{11}$

Hence, coordinates of point P are $(1,-1,4)$ or $\left(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11}\right)$

SECTION-E

36.



(i) $V = x^2 y \Rightarrow y = \frac{V}{x^2}$ ……… ….(i)

Hence, $S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}$

(ii) $\frac{dS}{dx} = 4\left(x - \frac{V}{x^2}\right)$

(iii) (a) $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2 y = x^3 \Rightarrow y = x$

$\frac{d^2S}{dx^2} = 4\left(1 + \frac{2V}{x^3}\right) > 0 \Rightarrow S$ is minimum if $y = x$.

OR

(iii) (b) $V = \frac{1}{4}(Sx - 2x^3) \Rightarrow \frac{dV}{dx} = \frac{1}{4}(S - 6x^2)$

Put $\frac{dV}{dx} = 0 \Rightarrow x = \sqrt{\frac{S}{6}}$

$\left(\frac{d^2V}{dx^2}\right)_{x=\sqrt{\frac{S}{6}}} = -3\sqrt{\frac{S}{6}} < 0 \Rightarrow$ Volume is maximum for $x = \sqrt{\frac{S}{6}}$.

37. **(i)** No, f is not bijective function

(ii) Range $= \{1,2,3,4, \ldots\ldots\ldots\ldots, 30\}$ and codomain $= N$

Since, Range $\neq$ codomain $\Rightarrow$ **f** is not onto and hence **f** is not bijective.

(iii) (a) $R = \{(1,3), (2,6), (3,9), (4,12), (5,15), (6,18), (7,21), (8,24), (9,27), (10,30)\}$

Since $(1,1) \notin R \Longrightarrow R$ is not reflexive.

$(1,3) \in R$ but $(3,1) \notin R \Longrightarrow R$ is not symmetric $(1,3) \in R, (3,9) \in R$ but $(1,9) \notin R \Longrightarrow R$ is not transitive.

OR

(iii) (b) $R = \{(\mathbf{1}, \mathbf{1}), (\mathbf{2}, \mathbf{8}), (\mathbf{3}, \mathbf{27})\}$

elements 4,5,6 … 30 do not have an image. Hence the above relation is not a function.

38. Let A: Event that chosen seed germinates.

B: Event that Brinjal seed is chosen.

C: Event that Cabbage seed is chosen.

R: Event that Radish seed is chosen.

$P(B)=\frac{10}{30};P(C)=\frac{12}{30};P(R)=\frac{8}{30};$

$P\left(\frac{A}{B}\right)=\frac{25}{100};P\left(\frac{A}{C}\right)=\frac{35}{100};P\left(\frac{A}{R}\right)=\frac{40}{100}$

(i) $P(A)=P(B)\cdot P\left(\frac{A}{B}\right)+P(C)\cdot P\left(\frac{A}{C}\right)+P(R)\cdot P\left(\frac{A}{R}\right)$

$=\frac{10}{30}\times\frac{25}{100}+\frac{12}{30}\times\frac{35}{100}+\frac{8}{30}\times\frac{40}{100}$

$=\frac{990}{3000}$ or $\frac{33}{100}$

(ii) $P\left(\frac{C}{A}\right)=\frac{P(C)\cdot P\left(\frac{A}{C}\right)}{P(B)\cdot P\left(\frac{A}{B}\right)+P(C)\cdot P\left(\frac{A}{C}\right)+P(R)\cdot P\left(\frac{A}{R}\right)}$

$=\frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$

$=\frac{42}{99}$ or $\frac{14}{33}$