CHAPTER-12 HERON's FORMULA

Exercise **10.1**

Question 1: A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side '*a*'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Solution: Side of traffic signal board = a

Perimeter of traffic signal board = $3 \times a$

$$2s = 3a \Rightarrow s = \frac{3}{2}a$$

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of given triangle} = \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$

$$= \sqrt{\left(\frac{3}{2}a\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$$

$$= \frac{\sqrt{3}}{4}a^2 \qquad \ldots (1)$$

Perimeter of traffic signal board = 180 cm

Side of traffic signal board

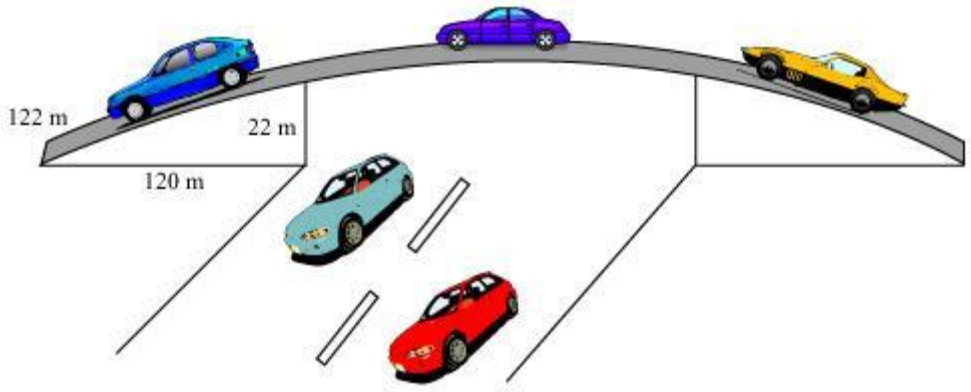
$$(a) = \left(\frac{180}{3}\right)\text{cm} = 60\,\text{cm}$$

Using equation (1), area of traffic signal board

$$= \frac{\sqrt{3}}{4}(60\text{ cm})^2$$

$$= \left(\frac{3600}{4}\sqrt{3}\right)\text{cm}^2 = 900\sqrt{3}\text{ cm}^2$$

Question 2: The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122m, 22m, and 120m (see the given figure). The advertisements yield an earning of Rs 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?



Solution: The sides of the triangle (i.e., *a*, *b*, *c*) are of 122 m, 22 m, and 120 m respectively.

Perimeter of triangle = (122 + 22 + 120) m

2*s* = 264 m

s = 132 m

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of given triangle} = \left[\sqrt{132(132-122)(132-22)(132-120)}\right] \text{m}^2$$

$$= \left[\sqrt{132(10)(110)(12)}\right] \text{m}^2 = 1320 \text{ m}^2$$

Rent of 1 m^2 area per year = Rs 5000

Rent of 1 m^2 area per month = Rs 5000/12

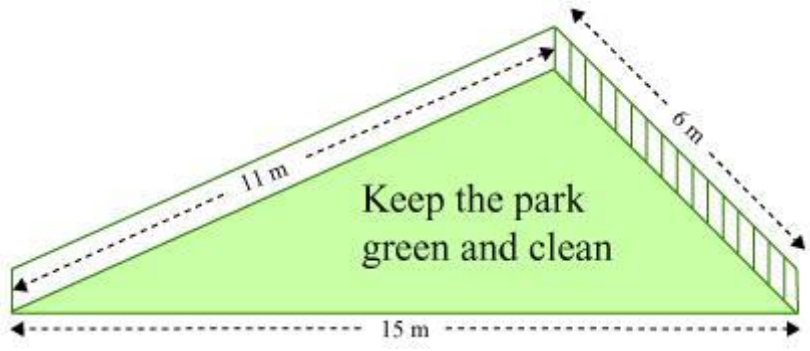
Rent of 1320 m^2 area for 3 months =

$$\text{Rs}\left(\frac{5000}{12} \times 3 \times 1320\right)$$

= Rs (5000 × 330) = Rs 1650000

Therefore, the company had to pay Rs 1650000.

Question 3: There is a slide in the park. One of its side walls has been painted in the same colour with a message "KEEP THE PARK GREEN AND CLEAN" (see the given figure). If the sides of the wall are 15m, 11m, and 6m, find the area painted in colour.



Solution: It can be observed that the area to be painted in colour is a triangle, having its sides as 11 m, 6 m, and 15 m.

Perimeter of such a triangle = (11 + 6 + 15) m

2 s = 32 m

s = 16 m

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \left[\sqrt{16(16-11)(16-6)(16-15)}\right] \text{m}^2$$
$$= \left(\sqrt{16\times5\times10\times1}\right) \text{m}^2$$
$$= 20\sqrt{2}\ \text{m}^2$$

Therefore, the area painted in colour is $20\sqrt{2}$ m^2.

Question 4: Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Solution: Let the third side of the triangle be x.

Perimeter of the given triangle = 42 cm

18 cm + 10 cm + x = 42

x = 14 cm

$$s = \frac{\text{Perimeter}}{2} = \frac{42 \text{ cm}}{2} = 21 \text{ cm}$$

By Heron's formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area of the given triangle} = \left(\sqrt{21(21-18)(21-10)(21-14)}\right)\text{cm}^2$$

$$= \left(\sqrt{21(3)(11)(7)}\right)\text{cm}^2$$

$$= 21\sqrt{11}\ \text{cm}^2$$

Question 5: Sides of a triangle are in the ratio of 12: 17: 25 and its perimeter is 540 cm. Find its area.

Solution: Let the common ratio between the sides of the given triangle be *x*.

Therefore, the side of the triangle will be 12*x*, 17*x*, and 25*x*.

Perimeter of this triangle = 540 cm

$12x + 17x + 25x = 540$ cm

$54x = 540$ cm

$x = 10$ cm

Sides of the triangle will be 120 cm, 170 cm, and 250 cm.

$$s = \frac{\text{Perimeter of triangle}}{2} = \frac{540\ \text{cm}}{2} = 270\ \text{cm}$$

By Heron's formula,

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \left[\sqrt{270(270-120)(270-170)(270-250)}\right]\text{cm}^2$$

$$= \left[\sqrt{270\times150\times100\times20}\right]\text{cm}^2$$

$$= 9000\ \text{cm}^2$$

Therefore, the area of this triangle is 9000 cm^2.

Question 6: An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Solution: Let the third side of this triangle be *x*.

Perimeter of triangle = 30 cm

12 cm + 12 cm + *x* = 30 cm

$x = 6$ cm

$$s = \frac{\text{Perimeter of triangle}}{2} = \frac{30 \text{ cm}}{2} = 15 \text{ cm}$$

By Heron's formula,

$$\begin{aligned}\text{Area of triangle} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \left[\sqrt{15(15-12)(15-12)(15-6)}\right] \text{cm}^2 \\ &= \left[\sqrt{15(3)(3)(9)}\right] \text{cm}^2 \\ &= 9\sqrt{15} \text{ cm}^2\end{aligned}$$