CHAPTER-1 NUMBER SYSTEM

Exercise 1.1

Question 1: Is zero a rational number? Can you write it in the form p/q, where p and q are integers and q ≠ 0?

Solution: Yes. Zero is a rational number as it can be represented as $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ etc.

Question 2: Find six rational numbers between 3 and 4.

Solution: There are infinite rational numbers between 3 and 4.

3 and 4 can be represented as 24/8 and 32/8 respectively.

Therefore, six rational numbers between 3 and 4 are

$$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$$

Question 3: Find five rational numbers between 3/5 and 4/5.

Solution: There are infinite rational numbers between 3/5 and 4/5.

$$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$$
$$\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

Therefore, rational numbers between 3/5 and 4/5 are

$$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$$

Question 4: State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Solution: (i) True; since the collection of whole numbers contains all natural numbers.

(ii) False; as integers may be negative but whole numbers are positive. For example: −3 is an integer but not a whole number.

(iii) False; as rational numbers may be fractional but whole numbers may not be. For example: 1/5 is a rational number but not a whole number.

Exercise 1.2

Question 1: State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.

(iii) Every real number is an irrational number.

Solution: (i) True; since the collection of real numbers is made up of rational and irrational numbers.

(ii) False; as negative numbers cannot be expressed as the square root of any other number.

(iii) False; as real numbers include both rational and irrational numbers. Therefore, every real number cannot be an irrational number.

Question 2: Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution: If numbers such as $\sqrt{4} = 2, \sqrt{9} = 3$ are considered.

Then here, 2 and 3 are rational numbers. Thus, the square roots of all positive integers are not irrational.

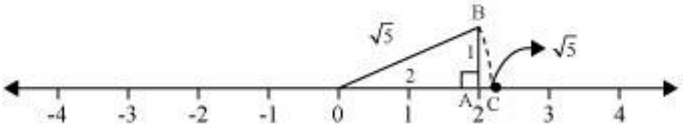
Question 3: Show how $\sqrt{5}$ can be represented on the number line.

Solution: We know that,

$\sqrt{4} = 2$

and,

$\sqrt{5} = \sqrt{(2)^2 + (1)^2}$



Mark a point 'A' representing 2 on number line. Now, construct AB of unit length perpendicular to OA. Then, taking O as centre and OB as radius, draw

an arc intersecting number line at C.

C is representing $\sqrt{5}$.

Exercise 1.3

Question 1: Write the following in decimal form and say what kind of decimal expansion each has:

(i) 36/100

(ii) 1/11

(iii) $\mathbf{4\frac{1}{8}}$

(iv) 3/13

(v) 2/11

(vi) 329/400

Solution: (i) $\frac{36}{100} = 0.36$

Terminating

(ii) $\frac{1}{11} = 0.090909 \ldots \ldots = 0.\overline{09}$

Non-terminating repeating

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating

(iv) $\frac{3}{13} = 0.230769230769 \ldots = 0.\overline{230769}$

Non-terminating repeating

(v) $\frac{2}{11} = 0.18181818 \ldots \ldots \ldots = 0.\overline{18}$

Non-terminating repeating

(vi) $\frac{329}{400} = 0.8225$

Terminating

Question 2: You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansion of 2/7, 3/7, 4/7, 5/7, 6/7are, without actually doing the long division? If so, how?

Solution: Yes. It can be done as follows.

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

Question 3: Express the following in the form p/q, where *p* and *q* are integers and *q* ≠ 0:

(i) $0.\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.\overline{001}$

Solution: (i) $0.\overline{6} = 0.666 \ldots$

Let $x = 0.666\ldots$

$10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$$x = \frac{2}{3}$$

(ii) $0.4\overline{7} = 0.4777 \ldots$

$$= \frac{4}{10} + \frac{0.777}{10}$$

Let $x = 0.777\ldots$

$10x = 7.777\ldots$

$10x = 7 + x$

$$x = \frac{7}{9}$$

$$\frac{4}{10} + \frac{0.777\ldots}{10} = \frac{4}{10} + \frac{7}{90}$$
$$= \frac{36+7}{90} = \frac{43}{90}$$

(iii) $0.\overline{001} = 0.001001 \ldots$

Let $x = 0.001001\ldots$

$1000x = 1.001001\ldots$

$1000x = 1 + x$

$999x = 1$

$$x = \frac{1}{999}$$

Question 4: Express 0.99999…in the form p/q. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution: Let $x = 0.9999\ldots$

$10x = 9.9999\ldots$

$10x = 9 + x$

$9x = 9$

$x = 1$

Question 5: What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.

Solution: It can be observed that,

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of 1/17.

Question 6: Look at several examples of rational numbers in the form p/q ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution: Terminating decimal expansion will occur when denominator *q* of rational number p/q is either of 2, 4, 5, 8, 10, and so on…

$\frac{9}{4} = 2.25$

$\frac{11}{8} = 1.375$

$\frac{27}{5} = 5.4$

It can be observed that terminating decimal may be obtained in the situation where prime factorisation of the denominator of the given fractions has the power of 2 only or 5 only or both.

Question 7: Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution: 3 numbers whose decimal expansions are non-terminating non-recurring are as follows.

0.505005000500005000005…

0.7207200720007200007200000…

0.08008000800008000008000008…

Question 8: Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Solution: $\frac{5}{7} = 0.\overline{714285}$ and

$\frac{9}{11} = 0.\overline{81}$

Therefore, 3 irrational numbers **between 5/7 and 9/11** are as follows.

0.73073007300073000073…

0.75075007500075000075…

0.79079007900079000079…

Question 9: Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$ (iii) 0.3796

(iv) 7.478478 (v) 1.101001000100001…

Solution: (i) $\sqrt{23} = 4.79583152331$ ...

As the decimal expansion of this number is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $\sqrt{225} = 15 = \frac{15}{1}$

It is a rational number as it can be represented in p/q form.

(iii) 0.3796

As the decimal expansion of this number is terminating, therefore, it is a rational number.

(iv) $7.478478 \ldots = 7.\overline{478}$

As the decimal expansion of this number is non-terminating recurring, therefore, it is a rational number.

(v) $1.10100100010000 \ldots$

As the decimal expansion of this number is non-terminating non-repeating, therefore, it is an irrational number.

Exercise 1.4

Question 1: Classify the following numbers as rational or irrational:

(i) $\mathbf{2 - \sqrt{5}}$

(ii)$\mathbf{(3 + \sqrt{23}) - \sqrt{23}}$

(iii) $\mathbf{\frac{2\sqrt{7}}{7\sqrt{7}}}$

(iv) $\mathbf{\frac{1}{\sqrt{2}}}$

(v) $\mathbf{2\pi}$

Solution:(i) $2 - \sqrt{5} = 2 - 2.2360679 \ldots$

$= -0.2360679 \ldots$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3 = \frac{3}{1}$

As it can be represented in p/q form, therefore, it is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$

As it can be represented in p/q form, therefore, it is a rational number.

(iv) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071067811 \ldots$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

(v) $2\pi = 2(3.1415 \ldots) = 6.2830 \ldots$

As the decimal expansion of this expression is non-terminating non-recurring, therefore, it is an irrational number.

Question 2: Simplify each of the following expressions:
(i) $(\mathbf{3}+\sqrt{\mathbf{3}})(\mathbf{2}+\sqrt{\mathbf{2}})$
$(\mathbf{ii})(\mathbf{3}+\sqrt{\mathbf{3}})(\mathbf{3}-\sqrt{\mathbf{3}})$
(iii) $(\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}})^{\mathbf{2}}$
(iv) $(\sqrt{\mathbf{5}}-\sqrt{\mathbf{2}})(\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}})$
Solution:

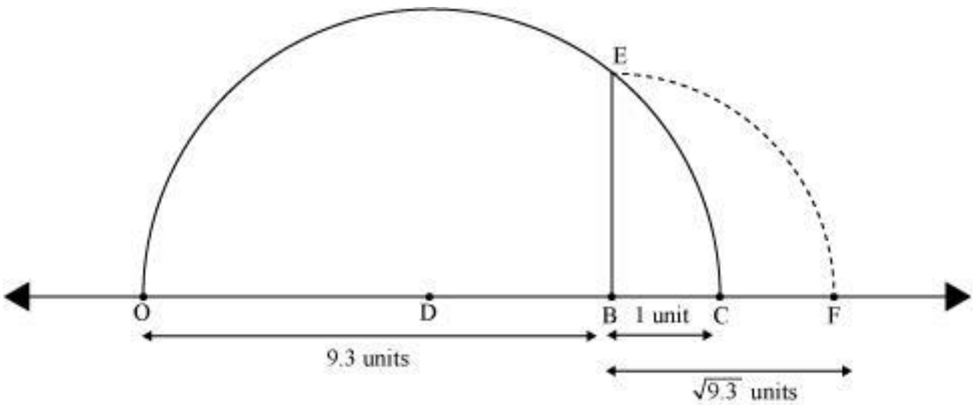
(i) $(3+\sqrt{3})(2+\sqrt{2}) = 3(2+\sqrt{2})+\sqrt{3}(2+\sqrt{2})$
$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$
(ii) $(3+\sqrt{3})(3-\sqrt{3}) = (3)^2-(\sqrt{3})^2$
$= 9-3 = 6$
(iii) $(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})$
$= 5+2+2\sqrt{10} = 7+2\sqrt{10}$
(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2-(\sqrt{2})^2$
$= 5-2 = 3$

Question 3: Recall, π is defined as the ratio of the circumference (say *c*) of a circle to its diameter (say *d*). That is, $\boldsymbol{\pi} = \frac{\boldsymbol{c}}{\boldsymbol{d}}$**. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?**

Solution: There is no contradiction. When we measure a length with scale or any other instrument, we only obtain an approximate rational value. We never obtain an exact value. For this reason, we may not realise that either *c* or *d* is irrational. Therefore, the fraction c/d is irrational. Hence, π is irrational.

Question 4: Represent $\sqrt{\mathbf{9.3}}$ **on the number line.**

Solution: Mark a line segment OB = 9.3 on number line. Further, take BC of 1 unit. Find the mid-point D of OC and draw a semi-circle on OC while taking D as its centre. Draw a perpendicular to line OC passing through point B. Let it intersect the semi-circle at E. Taking B as centre and BE as radius, draw an arc intersecting number line at F. BF is $\sqrt{9.3}$.



Question 5: Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Solution: (i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \frac{\sqrt{7}+\sqrt{6}}{1} = \sqrt{7}+\sqrt{6}$

(iii)

$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv)

$\frac{1}{\sqrt{7}-2} = \frac{(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$

$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

$= \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$

Exercise 1.5

Question 1: Find:

(i) $64^{\frac{1}{2}}$

(ii) $32^{\frac{1}{5}}$

(iii) $125^{\frac{1}{3}}$

Solution:

(i) $64^{\frac{1}{2}} = (2^6)^{\frac{1}{2}}$

$= 2^{6\times\frac{1}{2}}$

$= 2^3 = 8$

$[(a^m)^n = a^{mn}]$

(ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}}$

$= (2)^{5\times\frac{1}{5}}$ $[(a^{m\prime})^n = a^{mn}]$

$= 2^1 = 2$

(iii) $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}}$

$= 5^{3\times\frac{1}{3}}$ $[(a^m)^n = a^{mm}]$

$= 5^1 = 5$

Question 2: Find:

(i) $9^{\frac{3}{2}}$

(ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Solution:

(i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}}$

$= 3^{2\times\frac{3}{2}}$ $[(a^m)^n = a^{mmn}]$

$= 3^3 = 27$

(ii) $(32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}}$

$= 2^{5\times\frac{2}{5}}$ $[(a^m)^n = a^{mn}]$

$= 2^2 = 4$

(iii) $\quad (16)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}}$

$= 2^{4\times\frac{3}{4}}\,[(a^m)^n = a^{mm}]$

$= 2^3 = 8$

(iv)$(125)^{\frac{-1}{3}} \quad = \dfrac{1}{(125)^{\frac{1}{3}}} \qquad \left[a^{-m} = \dfrac{1}{a^m}\right]$

$= \dfrac{1}{(5^3)^{\frac{1}{3}}}$

$= \dfrac{1}{5^{3\times\frac{1}{3}}} \qquad [(a^m)^n = a^{m+n}]$

$= \dfrac{1}{5}$

Question 3: Simplify:

(i) $\mathbf{2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}}$

(ii) $\mathbf{\left(\frac{1}{3^3}\right)^7}$

(iii) $\mathbf{\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}}$

(iv) $\mathbf{7^{\frac{1}{2}}. 8^{\frac{1}{2}}}$

Solution:

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} \quad = 2^{\frac{2}{3}+\frac{1}{5}} \qquad [a^m \cdot a^n = a^{m+n}]$

$= 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

$(ii) \left(\dfrac{1}{3^3}\right)^7 \quad = \dfrac{1}{3^{3\times 7}} \quad [(a^m)^n = a^{mm}]$

$= \dfrac{1}{3^{21}} \qquad \left[\dfrac{1}{a^{m\prime}} = a^{-m}\right]$

$(iii) \quad \dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}\ \left[\dfrac{a^m}{a^n} = a^{m-n}\right]$

$= 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} \quad = (7 \times 8)^{\frac{1}{2}}\,[a^m \cdot b^m = (ab)^m]$

$= (56)^{\frac{1}{2}}$