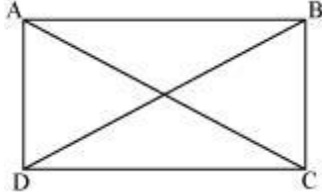


CHAPTER-8 QUADRILATERALS

Exercise 8.1

Question 1: If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Solution: Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90° .

In $\triangle ABC$ and $\triangle DCB$,

$AB = DC$ (Opposite sides of a parallelogram are equal)

$BC = BC$ (Common)

$AC = DB$ (Given)

$\therefore \triangle ABC \cong \triangle DCB$ (By SSS Congruence rule)

$\Rightarrow \angle ABC = \angle DCB$

It is known that the sum of the measures of angles on the same side of transversal is 180° .

$\angle ABC + \angle DCB = 180^\circ$ ($AB \parallel CD$)

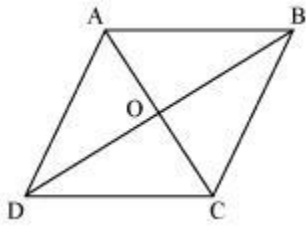
$\Rightarrow \angle ABC + \angle ABC = 180^\circ$

$\Rightarrow 2\angle ABC = 180^\circ$

$\Rightarrow \angle ABC = 90^\circ$

Since ABCD is a parallelogram and one of its interior angles is 90° , ABCD is a rectangle.

Question 2: Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.



Solution: Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e., $OA = OC$, $OB = OD$, and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$. To prove ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.

In $\triangle AOD$ and $\triangle COD$,

$OA = OC$ (Diagonals bisect each other)

$\angle AOD = \angle COD$ (Given)

$OD = OD$ (Common)

$\therefore \triangle AOD \cong \triangle COD$ (By SAS congruence rule)

$\therefore AD = CD$ (1)

Similarly, it can be proved that

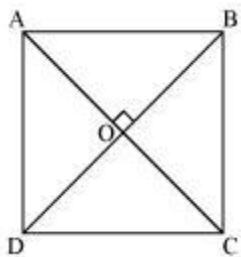
$AD = AB$ and $CD = BC$ (2)

From equations (1) and (2),

$AB = BC = CD = AD$

Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

Question 3: Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Solution: Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O. It is given that the diagonals of ABCD are equal and bisect each other at

right angles. Therefore, $AC = BD$, $OA = OC$, $OB = OD$, and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$. To prove ABCD is a square, we have to prove that ABCD is a parallelogram, $AB = BC = CD = AD$, and one of its interior angles is 90° .

In $\triangle AOB$ and $\triangle COD$,

$AO = CO$ (Diagonals bisect each other)

$OB = OD$ (Diagonals bisect each other)

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\therefore \triangle AOB \cong \triangle COD$ (SAS congruence rule)

$\therefore AB = CD$ (By CPCT) ... (1)

And, $\angle OAB = \angle OCD$ (By CPCT)

However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel.

$\therefore AB \parallel CD$... (2)

From equations (1) and (2), we obtain

ABCD is a parallelogram.

In $\triangle AOD$ and $\triangle COD$,

$AO = CO$ (Diagonals bisect each other)

$\angle AOD = \angle COD$ (Given that each is 90°)

$OD = OD$ (Common)

$\therefore \triangle AOD \cong \triangle COD$ (SAS congruence rule)

$\therefore AD = DC$... (3)

However, $AD = BC$ and $AB = CD$ (Opposite sides of parallelogram ABCD)

$\therefore AB = BC = CD = DA$

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In $\triangle ADC$ and $\triangle BCD$,

$AD = BC$ (Already proved)

$AC = BD$ (Given)

$DC = CD$ (Common)

$\therefore \triangle ADC \cong \triangle BCD$ (SSS Congruence rule)

$\therefore \angle ADC = \angle BCD$ (By CPCT)

However, $\angle ADC + \angle BCD = 180^\circ$ (Co-interior angles)

$$\Rightarrow \angle ADC + \angle ADC = 180^\circ$$

$$\Rightarrow 2\angle ADC = 180^\circ$$

$$\Rightarrow \angle ADC = 90^\circ$$

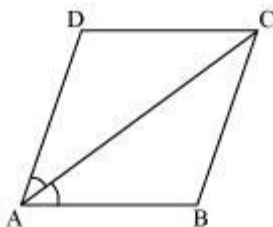
One of the interior angles of quadrilateral ABCD is a right angle.

Thus, we have obtained that ABCD is a parallelogram, $AB = BC = CD = AD$ and one of its interior angles is 90° . Therefore, ABCD is a square.

Question 4: Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see the given figure). Show that

(i) It bisects $\angle C$ also,

(ii) ABCD is a rhombus.



Solution: (i) ABCD is a parallelogram.

$$\therefore \angle DAC = \angle BCA \text{ (Alternate interior angles) ... (1)}$$

$$\text{And, } \angle BAC = \angle DCA \text{ (Alternate interior angles) ... (2)}$$

However, it is given that AC bisects $\angle A$.

$$\therefore \angle DAC = \angle BAC \text{ ... (3)}$$

From equations (1), (2), and (3), we obtain

$$\angle DAC = \angle BCA = \angle BAC = \angle DCA \text{ ... (4)}$$

$$\Rightarrow \angle DCA = \angle BCA$$

Hence, AC bisects $\angle C$.

(ii) From equation (4), we obtain

$$\angle DAC = \angle DCA$$

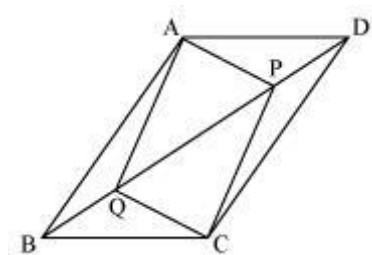
$\therefore DA = DC$ (Side opposite to equal angles are equal)

However, $DA = BC$ and $AB = CD$ (Opposite sides of a parallelogram)

$$\therefore AB = BC = CD = DA$$

Hence, ABCD is a rhombus.

Question 5: In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$ (see the given figure). Show that:



(i) $\triangle APD \cong \triangle CQB$

(ii) $AP = CQ$

(iii) $\triangle AQB \cong \triangle CPD$

(iv) $AQ = CP$

(v) APCQ is a parallelogram

Solution: (i) In $\triangle APD$ and $\triangle CQB$,

$\angle ADP = \angle CBQ$ (Alternate interior angles for $BC \parallel AD$)

$AD = CB$ (Opposite sides of parallelogram ABCD)

$DP = BQ$ (Given)

$\therefore \triangle APD \cong \triangle CQB$ (Using SAS congruence rule)

(ii) As we had observed that $\triangle APD \cong \triangle CQB$,

$\therefore AP = CQ$ (CPCT)

(iii) In $\triangle AQB$ and $\triangle CPD$,

$\angle ABQ = \angle CDP$ (Alternate interior angles for $AB \parallel CD$)

$AB = CD$ (Opposite sides of parallelogram ABCD)

$BQ = DP$ (Given)

$\therefore \triangle AQB \cong \triangle CPD$ (Using SAS congruence rule)

(iv) As we had observed that $\triangle AQB \cong \triangle CPD$,

$\therefore AQ = CP$ (CPCT)

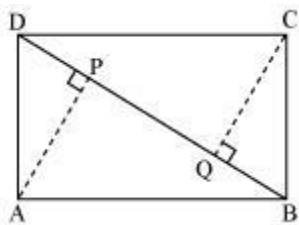
(v) From the result obtained in (ii) and (iv),

$AQ = CP$ and

$AP = CQ$

Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

Question 6: ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



(i) $\triangle APB \cong \triangle CQD$

(ii) $AP = CQ$

Solution: (i) In $\triangle APB$ and $\triangle CQD$,

$\angle APB = \angle CQD$ (Each 90°)

$AB = CD$ (Opposite sides of parallelogram ABCD)

$\angle ABP = \angle CDQ$ (Alternate interior angles for $AB \parallel CD$)

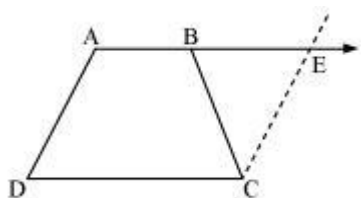
$\therefore \triangle APB \cong \triangle CQD$ (By AAS congruency)

(ii) By using the above result

$\triangle APB \cong \triangle CQD$, we obtain

$AP = CQ$ (By CPCT)

Question 7: ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see the given figure). Show that



(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) diagonal $AC =$ diagonal BD

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Solution: Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting AE at point E. It is clear that AECD is a parallelogram.

(i) $AD = CE$ (Opposite sides of parallelogram AECD)

However, $AD = BC$ (Given)

Therefore, $BC = CE$

$\angle CEB = \angle CBE$ (Angle opposite to equal sides are also equal)

Consider parallel lines AD and CE. AE is the transversal line for them.

$\angle A + \angle CEB = 180^\circ$ (Angles on the same side of transversal)

$\angle A + \angle CBE = 180^\circ$ (Using the relation $\angle CEB = \angle CBE$) ... (1)

However, $\angle B + \angle CBE = 180^\circ$ (Linear pair angles) ... (2)

From equations (1) and (2), we obtain

$\angle A = \angle B$

(ii) $AB \parallel CD$

$\angle A + \angle D = 180^\circ$ (Angles on the same side of the transversal)

Also, $\angle C + \angle B = 180^\circ$ (Angles on the same side of the transversal)

$$\therefore \angle A + \angle D = \angle C + \angle B$$

However, $\angle A = \angle B$ [Using the result obtained in (i)]

$$\therefore \angle C = \angle D$$

(iii) In $\triangle ABC$ and $\triangle BAD$,

$AB = BA$ (Common side)

$BC = AD$ (Given)

$\angle B = \angle A$ (Proved before)

$\therefore \triangle ABC \cong \triangle BAD$ (SAS congruence rule)

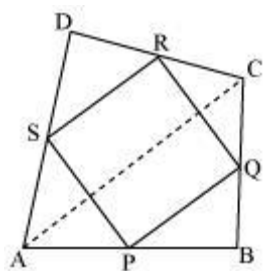
(iv) We had observed that,

$$\triangle ABC \cong \triangle BAD$$

$\therefore AC = BD$ (By CPCT)

Exercise 8.2

Question 1: ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.

Solution: (i) In $\triangle ADC$, S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \dots (1)$$

(ii) In $\triangle ABC$, P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \dots (2)$$

Using equations (1) and (2), we obtain

$$PQ \parallel SR \text{ and } PQ = SR \dots (3)$$

$$\Rightarrow PQ = SR$$

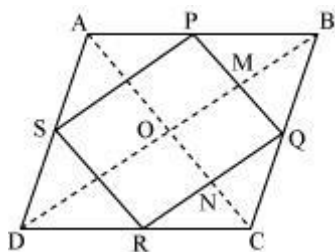
(iii) From equation (3), we obtained

$$PQ \parallel SR \text{ and } PQ = SR$$

Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

Hence, PQRS is a parallelogram.

Question 2: ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.



Solution: In $\triangle ABC$, P and Q are the mid-points of sides AB and BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \text{ (Using mid-point theorem)} \dots (1)$$

In $\triangle ADC$,

R and S are the mid-points of CD and AD respectively.

$$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2} AC \text{ (Using mid-point theorem)} \dots (2)$$

From equations (1) and (2), we obtain

$$PQ \parallel RS \text{ and } PQ = RS$$

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to

each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,

$MQ \parallel ON$ ($\because PQ \parallel AC$)

$QN \parallel OM$ ($\because QR \parallel BD$)

Therefore, OMQN is a parallelogram.

$\Rightarrow \angle MQN = \angle NOM$

$\Rightarrow \angle PQR = \angle NOM$

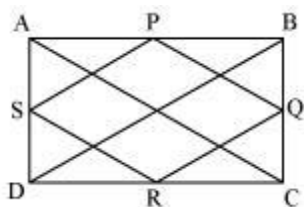
However, $\angle NOM = 90^\circ$ (Diagonals of a rhombus are perpendicular to each other)

$\therefore \angle PQR = 90^\circ$

Clearly, PQRS is a parallelogram having one of its interior angles as 90° .

Hence, PQRS is a rectangle.

Question 3: ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.



Solution: Let us join AC and BD.

In $\triangle ABC$,

P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ (Mid-point theorem) ... (1)

Similarly in $\triangle ADC$,

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ (Mid-point theorem) ... (2)

Clearly, $PQ \parallel SR$ and $PQ = SR$

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to

each other, it is a parallelogram.

$\therefore PS \parallel QR$ and $PS = QR$ (Opposite sides of parallelogram)... (3)

In $\triangle BCD$, Q and R are the mid-points of side BC and CD respectively.

$\therefore QR \parallel BD$ and $QR = \frac{1}{2} BD$ (Mid-point theorem) ... (4)

However, the diagonals of a rectangle are equal.

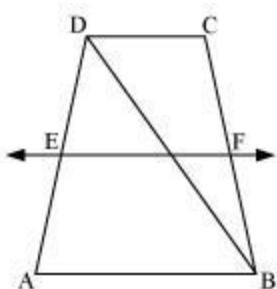
$\therefore AC = BD$... (5)

By using equation (1), (2), (3), (4), and (5), we obtain

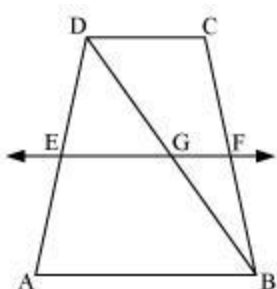
$$PQ = QR = SR = PS$$

Therefore, PQRS is a rhombus.

Question 4: ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see the given figure). Show that F is the mid-point of BC.



Solution: Let EF intersect DB at G.



By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

In $\triangle ABD$,

$EF \parallel AB$ and E is the mid-point of AD.

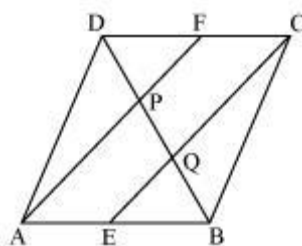
Therefore, G will be the mid-point of DB.

As $EF \parallel AB$ and $AB \parallel CD$,

$\therefore EF \parallel CD$ (Two lines parallel to the same line are parallel to each other)

In $\triangle BCD$, $GF \parallel CD$ and G is the mid-point of line BD. Therefore, by using converse of mid-point theorem, F is the mid-point of BC.

Question 5: In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



Solution: ABCD is a parallelogram.

$\therefore AB \parallel CD$

And hence, $AE \parallel FC$

Again, $AB = CD$ (Opposite sides of parallelogram ABCD)

$$\frac{1}{2} AB = \frac{1}{2} CD$$

$AE = FC$ (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

$\Rightarrow AF \parallel EC$ (Opposite sides of a parallelogram)

In $\triangle DQC$, F is the mid-point of side DC and $FP \parallel CQ$ (as $AF \parallel EC$). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

$$\Rightarrow DP = PQ \dots (1)$$

Similarly, in $\triangle APB$, E is the mid-point of side AB and $EQ \parallel AP$ (as $AF \parallel EC$). Therefore, by using the converse of mid-point theorem, it can be said that

Q is the mid-point of PB.

$$\Rightarrow PQ = QB \dots (2)$$

From equations (1) and (2),

$$DP = PQ = BQ$$

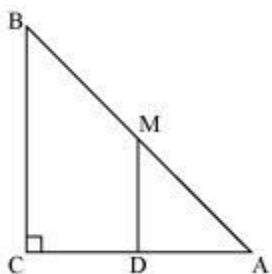
Hence, the line segments AF and EC trisect the diagonal BD.

Question 6: ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) $MD \perp AC$

(iii) $CM = MA = \frac{1}{2} AB$



Solution: (i) In $\triangle ABC$,

It is given that M is the mid-point of AB and $MD \parallel BC$.

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As $DM \parallel CB$ and AC is a transversal line for them, therefore,

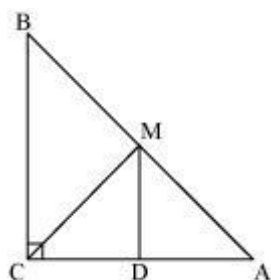
$$\angle MDC + \angle DCB = 180^\circ \text{ (Co-interior angles)}$$

$$\angle MDC + 90^\circ = 180^\circ$$

$$\angle MDC = 90^\circ$$

$$\therefore MD \perp AC$$

(iii) Join MC.



In $\triangle AMD$ and $\triangle CMD$,

$AD = CD$ (D is the mid-point of side AC)

$\angle ADM = \angle CDM$ (Each 90°)

$DM = DM$ (Common)

$\therefore \triangle AMD \cong \triangle CMD$ (By SAS congruence rule)

Therefore, $AM = CM$ (By CPCT)

However, $AM = \frac{1}{2} AB$ (M is the mid-point of AB)

Therefore, it can be said that

$$CM = AM = \frac{1}{2} AB$$